IMAGE EVALUATION
TEST TARGET (MT-3)

6"

Technical and Bibliographic Notes/Notes techniques et bibliographiques

The Institute has attempted to obtain the best original copy available for filming. Features of this copy which may be bibliographically unique, which may alter any of the images in the reproduction, or which may significantly change the usual method of filming, are checked below.

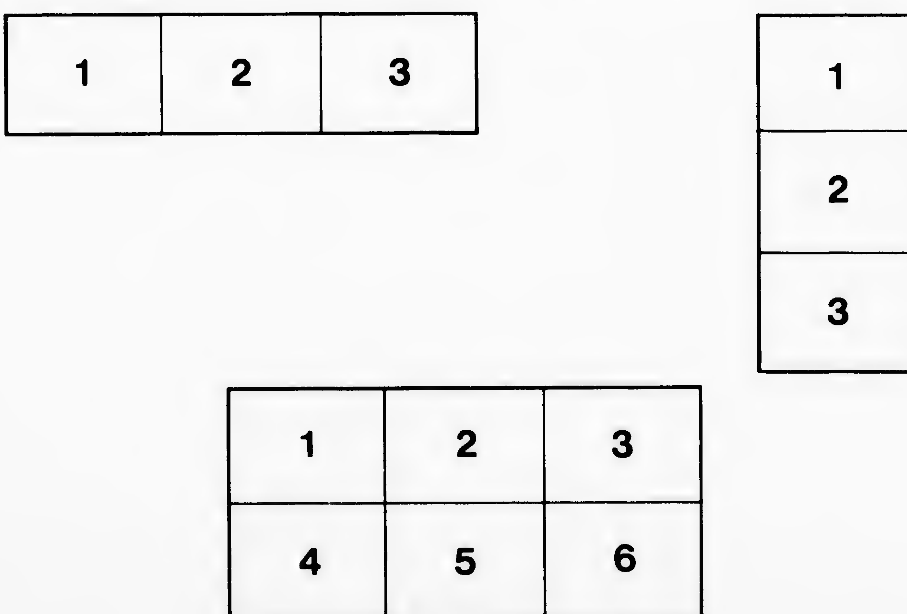
L'Institut a microfilmé le meilleur exemplaire qu'il lui a été possible de se procurer. Les détails de cet exemplaire qui sont peut-être uniques du point de vue bibliographique, qui peuvent modifier une image reproduite, ou qui peuvent exiger une modification dans la méthode normale de filmage sont indiqués ci-dessous.

- [ ] Coloured covers/ Couverture de couleur
- [ ] Covers damaged/ Couverture endommagée
- [ ] Covers restored and/or laminated/ Couverture restaurée et/ou pelliculée
- [ ] Cover title missing/ Le titre de couverture manque
- [ ] Coloured maps/ Cartes géographiques en couleur
- [ ] Coloured ink (i.e. other than blue or black)/ Encre de couleur (i.e. autre que bleue ou noire)
- [ ] Coloured plates and/or illustrations/ Planches et/ou illustrations en couleur
- [ ] Bound with other material/ Relié avec d'autres documents
- [ ] Tight binding may cause shadows or distortion along interior margin/ La reliure serrée peut causer de l'ombre ou de la distortion le long de la marge intérieure
- [ ] Blank leaves added during restoration may appear within the text. Whenever possible, these have been omitted from filming/ Il se peut que certaines pages blanches ajoutées lors d'une restauration apparaissent dans le texte, mais, lorsque cela était possible, ces pages n'ont pas été filmées.
- [ ] Additional comments:/ Commentaires supplémentaires:

- [ ] Coloured pages/ Pages de couleur
- [ ] Pages damaged/ Pages endommagées
- [ ] Pages restored and/or laminated/ Pages restaurées et/ou pelliculées
- [x] Pages discoloured, stained or foxed/ Pages décolorées, tachetées ou piquées
- [ ] Pages detached/ Pages détachées
- [x] Showthrough/ Transparence
- [ ] Quality of print varies/ Qualité inégale de l'impression
- [ ] Includes supplementary material/ Comprend du matériel supplémentaire
- [ ] Only edition available/ Seule édition disponible
- [ ] Pages wholly or partially obscured by errata slips, tissues, etc., have been refilmed to ensure the best possible image/ Les pages totalement ou partiellement obscurcies par un feuillet d'errata, une pelure, etc., ont été filmées à nouveau de façon à obtenir la meilleure image possible.

This item is filmed at the reduction ratio checked below/
Ce document est filmé au taux de réduction indiqué ci-dessous.

10X	12X	14X	16X	18X	20X	22X	24X	26X	28X	30X	32X
								✓			


1
2
3
1
2
3
1
2
3
4
5
6

A TRANSFORMATION

OF

HANSEN'S LUNAR THEORY

COMPARED WITH THE

THEORY OF DELAUNAY.

BY

SIMON NEWCOMB,
SUPERINTENDENT AMERICAN EPHEMERIS,

AIDED BY

JOHN MEIER,
ASSISTANT AMERICAN EPHEMERIS.

TRANSFORMATION OF HANSEN'S LUNAR THEORY.

The numerical computation of the inequalities in the moon's motion executed by HANSEN was probably the greatest step taken in recent times toward placing the theory of the lunar perturbations on an accurate numerical basis. It was the step which first rendered it certain that any discrepancy between the theoretical and observed values of the inequalities produced by the sun arose from some other cause than errors in theory. The theoretical values to which it led must be considered the most accurate which astronomy now possesses.

The only theory which can compete with HANSEN's is that of DELAUNAY. Here the coefficients are developed in series converging so slowly that some of the results are still a little doubtful, notwithstanding the great extent to which the approximation was carried. It may be expected that the numerical theory on which Sir GEORGE AIRY is now engaged will form yet another step in advance, in which nothing will be wanting for the purposes of accurate astronomy, so that three theories of the highest order of accuracy will ultimately be available for the construction of lunar tables. The work in question being still unfinished, the results of HANSEN and DELAUNAY are the only ones now available.

Unfortunately, the theory of HANSEN cannot be directly compared with those which have preceded it, owing to the peculiar form of the variables in which the co-ordinates of the moon are expressed. In saying this, I do not contest the proposition that this form has advantages. But, apart from the question of its merits in form, it becomes important to have the means of making a direct comparison of HANSEN's theory with that of his predecessors and colaborers, who have expressed the co-ordinates of the moon directly in terms of the time. This has twice been partially done: by the writer in the *Comptes Rendus* for 1868, I (Tome LXVI, p. 1197), and, independently, by SCHJELLERUP, in a paper published in 1874 by the Danish Academy of Sciences. Both depend on data for the transformation given by HANSEN himself, which, though they may be accurate enough to give an idea of the agreement between the theories of HANSEN and DELAUNAY, cannot be regarded as sufficiently precise for a satisfactory transformed theory. The object of the present paper is to make a transformation which shall faithfully represent HANSEN's latest theory, and be expressed in arguments depending directly on the time.

§ 1.

EXPRESSION OF THE MOON'S LONGITUDE.

In HANSEN's theory the moon's longitude is represented in the following form. Put

- g, the moon's mean anomaly;
- g', the sun's mean anomaly;
- ω, the distance from the node to the perigee;
- ω', the distance from the node to the solar perigee;
- π, the longitude of the perigee;
- e, the eccentricity of the moon's orbit, as used by HANSEN;
- $n\delta z$, the Hansenian perturbations of mean anomaly;
- s, the Hansenian perturbations of latitude;
- I, the inclination of the moon's orbit.

Then put, as auxiliary quantities,

$$f = \text{elta}\,(e, g + n\delta z), \text{ the true anomaly;}$$

$$R = -\tan^2 \tfrac{1}{2} I \sin 2\,(f+\omega) + \tfrac{1}{2}\tan^4 \tfrac{1}{2} I \sin 4\,(f+\omega) - \text{etc.},$$

the reduction to the ecliptic;

$$\begin{aligned} R' = & -s\,\frac{\tan I \cos (f+\omega)}{1 - \sin^2 I \sin^2 (f+\omega)} \\ & - 0''.397 \sin 2\,\omega \\ & - 1''.198 \sin (2\,g' + 2\,\omega') \\ & - 0''.285 \sin (2\,g - 4\,g' + 2\,\omega - 4\,\omega'), \end{aligned}$$

the inequalities of this reduction.

Then, for the moon's longitude,

$$L = f + \pi + R + R'.$$

The latitude, β, is given by the equation

$$\sin \beta = \sin I \sin (f+\omega) + s.$$

In presenting HANSEN's results in the form of a complete and exact numerical theory, several precautions have to be taken. In the first place, all the results must, so far as possible, depend upon or be reduced to one and the same homogeneous set of elements. In the next place, those inequalities which express the solution of the problem of three bodies, considered as material points, must be separated from inequalities arising from other sources, such, for instance, as the distance between the moon's centres of gravity and figure, and the ellipticity of the earth.

Three values of the eccentricity appear in HANSEN's theory and tables:

(1) A provisional or ideal eccentricity, with which the inequalities were originally computed.

(2) An apparent eccentricity, which he found to represent the observed motion of the moon's centre of figure, and used in his tables.

(3) A theoretical eccentricity of the true orbit described by the moon's centre of gravity.

These three values of the element are:—

$$(1)\quad e = .05490079$$
$$(2)\quad e = .05490807$$
$$(3)\quad e = .05489959$$

According to HANSEN's view it is the third value which should be used in computing the moon's perturbations; but as he actually used the first value, it is the one which we should employ in the transformation.

In the case of the inclination there are three corresponding values, with an additional complication arising from the question whether we shall add to the inclination a term in the perturbations, $2''.705 \sin (g + \omega)$, having the mean argument of latitude as its argument.

Omitting this term, the values of the inclination will be:—

$$(1)\quad I = 5^\circ\ 8'\ 48''$$
$$(2)\quad I = 5^\circ\ 8'\ 43''.66$$
$$(3)\quad I = 5^\circ\ 8'\ 39''.96$$

Here, again, it is only the first value with which we are concerned in the transformation, because it is the one employed by HANSEN in computing the perturbations.

The Hansenian perturbation $n\delta z$ is an explicit function of g, g', ω and ω'. So far as the longitude is concerned, our present problem is to express f, R and R', and thence L, as explicit functions of the above four quantities. If we put:—

$$z = g + n\delta z$$

e_1, e_2, e_3, etc., the coefficients of $\sin z$, $\sin 2z$, etc. in the development of elta (e, z), we shall have,

$$f = z + e_1 \sin z + e_2 \sin 2z + \text{etc.}$$

If, then, we put $g + n\delta z$ for z, develop in powers of $n\delta z$, call $(e, g)_0$ the part of f independent of $n\delta z$, and $(e, g)_i$ the coefficient of $(n\delta z)^i$ in f, we shall have,

$$(e, g)_0 = g + e_1 \sin g + e_2 \sin 2g + e_3 \sin 3g + e_4 \sin 4g + \text{etc.}$$
$$(e, g)_1 = 1 + e_1 \cos g + 2e_2 \cos 2g + 3e_3 \cos 3g + \text{etc.}$$
$$(e, g)_2 = -\frac{1}{2} e_1 \sin g - \frac{2^2}{2} e_2 \sin 2g - \frac{3^2}{2} e_3 \sin 3g - \text{etc.}$$
$$(e, g)_3 = -\frac{1}{2.3} e_1 \cos g - \frac{2^3}{2.3} e_2 \cos 2g - \frac{3^3}{2.3} e_3 \cos 3g - \text{etc.}$$
$$(e, g)_4 = \frac{1}{2.3.4} e_1 \sin g + \text{etc.}$$

etc. etc.

The coefficients e_1, e_2, etc., are dependent on the eccentricity. The well-known analytical values, and the numerical values obtained by putting $e = .05490070$, are:

$$e_1 = 2e - \frac{1}{4} e^3 + \frac{5}{96} e^5 = .10976024 = 22639''.676$$
$$e_2 = \frac{5}{4} e^2 - \frac{11}{24} e^4 + \frac{17}{192} e^6 = .00376346 = 776''.269$$
$$e_3 = \frac{13}{12} e^3 - \frac{43}{64} e^5 = .00017893 = 36''.907$$
$$e_4 = \frac{103}{96} e^4 - \frac{451}{480} e^6 = .00000972 = 2''.005$$
$$e_5 = \frac{1097}{960} e^5 = .00000057 = 0''.118$$
$$e_6 = \frac{1223}{960} e^6 = .00000004 = 0''.007$$

The value of $n\delta z$ is taken, not from HANSEN's tables, but from his revised results given in the *Darlegung**. They are found in Part I, pp. 409–411, and Part II, pp. 224, 242, 258, and 268, and, for convenience of reference, are all collected in Table I of the present paper. In this table are given also the powers of $n\delta z$, the computations of which were all made in duplicate, that of the square being executed by two independent computers.

We thus have all the data for the numerical value of f, the formula for which is,

$$f = (e, g)_0 + (e, g)_1 \, n\delta z + (e, g)_2 \, (n\delta z)^2 + \text{etc.} \qquad (1)$$

Consider next the first term of R, which we may call R_1. We have

$$R_1 = -\tan^2 \frac{1}{2} I \sin (2f + 2\omega),$$

which is also to be developed in powers of $n\delta z$.

* Under this title reference is made to Hansen's two papers, *Darlegung der theoretischen Berechnung der in den Mondtafeln angewandten Störungen*, in the *Abhandlungen der königlich-sächsischen Gesellschaft der Wissenschaften. Band IX, XI.*

If we substitute for f its value in terms of e and z, and develop in powers of e, we find * :—

$$
R_1 = -\tan^2 \tfrac{1}{2} I \times \left\{
\begin{array}{rl}
\frac{1}{24} e^4 & \sin(-2z + 2\omega) \\
+ \frac{1}{12} e^3 & \sin(-z + 2\omega) \\
+ \left(\frac{3}{4} e^2 + \frac{1}{8} e^4\right) & \sin 2\omega \\
+ \left(-2e + \frac{7}{4} e^3\right) & \sin(z + 2\omega) \\
+ \left(1 - 4e^2 + \frac{55}{16} e^4\right) & \sin(2z + 2\omega) \\
+ \left(2e - \frac{27}{4} e^3\right) & \sin(3z + 2\omega) \\
+ \left(\frac{13}{4} e^2 - \frac{259}{24} e^4\right) & \sin(4z + 2\omega) \\
+ \frac{59}{12} e^3 & \sin(5z + 2\omega) \\
+ \frac{115}{16} e^4 & \sin(6z + 2\omega)
\end{array}
\right.
$$

If, in this equation, we substitute for e and I their numerical values and then differentiate with respect to z, so as to obtain the coefficients of the powers of $n\delta z$, putting

$$R_1 = R_{1,0} + R_{1,1}\, n\delta z + R_{1,2}\, (n\delta z)^2 + \text{etc.},$$

we have

$$
\begin{array}{rl}
R_{1,0} = - & 0''.006 \sin(-g + 2\omega) \\
- & 0''.942 \sin(2\omega) \\
+ & 45''.627 \sin(g + 2\omega) \\
- & 411''.626 \sin(2g + 2\omega) \\
- & 45''.281 \sin(3g + 2\omega) \\
- & 4''.040 \sin(4g + 2\omega) \\
- & 0''.338 \sin(5g + 2\omega) \\
- & 0''.027 \sin(6g + 2\omega)
\end{array}
$$

$$
\begin{array}{rl}
R_{1,1} = & +.000\,221{,}2 \cos(g + 2\omega) \\
& -.003\,991{,}2 \cos(2g + 2\omega) \\
& -.000\,658{,}6 \cos(3g + 2\omega) \\
& -.000\,078{,}3 \cos(4g + 2\omega) \\
& -.000\,008{,}2 \cos(5g + 2\omega) \\
& -.000\,000{,}8 \cos(6g + 2\omega)
\end{array}
$$

* Tables of this and the other developments in the elliptic motion have been given by Professor CAYLEY in the *Memoirs of the Royal Astronomical Society*, Vol. XXIX, but the above development was executed independently before the applicability of Professor CAYLEY's formulæ was remarked.

$$
\begin{aligned}
R_{1,2} = &- .00011 \sin(\ \ g + 2\,\omega) \\
&+ .00399 \sin(2\,g + 2\,\omega) \\
&+ .00099 \sin(3\,g + 2\,\omega) \\
&+ .00016 \sin(4\,g + 2\,\omega)
\end{aligned}
$$

$$
\begin{aligned}
R_{1,3} = &+ .0027 \cos(2\,g + 2\,\omega) \\
&+ .0010 \cos(3\,g + 2\,\omega)
\end{aligned}
$$

In the same way, putting

$$R_2 = \frac{1}{2} \tan^4 \frac{1}{2} I \sin(4\,f + 4\,\omega)$$

we have by substituting for f its value in z, and developing in powers of e,

$$
\begin{aligned}
\sin(4\,f + 4\,\omega) = &\quad \frac{11}{2} e^2 \sin(2\,z + 4\,\omega) \\
&- \quad 4\,e \quad \sin(3\,z + 4\,\omega) \\
&+ (1 - 16\,e^2) \sin(4\,z + 4\,\omega) \\
&+ \quad 4\,e \quad \sin(5\,z + 4\,\omega) \\
&+ \frac{21}{2} e^2 \sin(6\,z + 4\,\omega)
\end{aligned}
$$

Putting as before,

$$R_2 = R_{2,0} + R_{2,1}\, n\,\delta z + R_{2,2}\,(n\,\delta z)^2 + \text{etc.},$$

we find by substituting the numerical values of I and e

$$
\begin{aligned}
R_{2,0} = &+ 0''.007 \sin(2\,g + 4\,\omega) \\
&- 0''.092 \sin(3\,g + 4\,\omega) \\
&+ 0''.400 \sin(4\,g + 4\,\omega) \\
&+ 0''.092 \sin(5\,g + 4\,\omega) \\
&+ 0''.013 \sin(6\,g + 4\,\omega)
\end{aligned}
$$

$$
\begin{aligned}
R_{2,1} = &- .000\,001{,}3 \cos(3\,g + 4\,\omega) \\
&+ .000\,007{,}8 \cos(4\,g + 4\,\omega) \\
&+ .000\,002{,}2 \cos(5\,g + 4\,\omega)
\end{aligned}
$$

The terms of $R_{2,2}\,(n\,\delta z)^2$ are less than $0''.001$.

The coefficient of $-s \tan I$ in R' is, with sufficient accuracy,

$$\cos(f + \omega)\,[1 + \sin^2 I \sin^2(f + \omega)]$$

or

$$\left(1 + \frac{1}{4} \sin^2 I\right) \cos(f + \omega) - \frac{1}{4} \sin^2 I \cos(3\,f + 3\,\omega).$$

By the developments of the elliptic motion we have,

$$\begin{aligned}
\cos(f+\omega) = & -\frac{1}{12}e^3\cos(-2z+\omega) \\
& -\frac{1}{8}e^2\cos(-z+\omega) \\
& -e\cos\omega \\
& +(1-e^2)\cos(z+\omega) \\
& +(e-\frac{5}{4}e^3)\cos(2z+\omega) \\
& +\frac{9}{8}e^2\cos(3z+\omega) \\
& +\frac{4}{3}e^3\cos(4z+\omega).
\end{aligned}$$

$$\begin{aligned}
\cos(3f+3\omega) = & \ \frac{21}{8}e^2\cos(z+3\omega) \\
& -3e\cos(2z+3\omega) \\
& +(1-9e^2)\cos(3z+3\omega) \\
& +3e\cos(4z+3\omega) \\
& +\frac{51}{8}e^2\cos(5z+3\omega)
\end{aligned}$$

If we represent by S the coefficient of s in R′, that is,

$$S = -\tan I\cos(f+\omega)\{1+\sin^2 I\sin^2(f+\omega)\},$$

and suppose

$$S = S_0 + S_1\,n\delta z + S_2\,(n\delta z)^2,$$

we shall have,

$$\begin{aligned}
S_0 = & +.000034\cos(-g+\omega) \\
& +.004955\cos\omega \\
& -.089978\cos(g+\omega) \\
& -.004936\cos(2g+\omega) \\
& -.000306\cos(3g+\omega) \\
& -.000020\cos(4g+\omega) \\
& -.000030\cos(2g+3\omega) \\
& +.000176\cos(3g+3\omega) \\
& +.000030\cos(4g+3\omega) \\
& +.000004\cos(5g+3\omega)
\end{aligned}$$

$$\begin{aligned}
S_1 = & +.0900\sin(g+\omega) \\
& +.0099\sin(2g+\omega) \\
& +.0009\sin(3g+\omega)
\end{aligned}$$

$$\begin{aligned}
S_2 = & +.045\cos(g+\omega) \\
& +.010\cos(2g+\omega)
\end{aligned}$$

Multiplying these several expressions by HANSEN'S s, we find the value of $s\,S_0$, etc., given in Table II.

Collecting all the coefficients of the powers of $n\delta z$, we find the following expressions for the moon's true ecliptic longitude, as a function of $n\delta z$:—

$$L = L_0 + L_1 n\delta z + L_2 (n\delta z)^2 + \text{etc.}$$

Terms independent of $n\delta z$.

$$
\begin{aligned}
L_0 = g + \pi & \\
& + 22639''.676 \sin g \\
& + 776''.269 \sin 2g \\
& + 36''.907 \sin 3g \\
& + 2''.005 \sin 4g \\
& + 0''.118 \sin 5g \\
& + 0''.007 \sin 6g \\
& - 0''.006 \sin (-g + 2\omega) \\
& \left\{\begin{matrix} - 0''.942 \\ - 0''.397 \end{matrix}\right\} \sin 2\omega \\
& + 45''.627 \sin (g + 2\omega) \\
& - 411''.626 \sin (2g + 2\omega) \\
& - 45''.281 \sin (3g + 2\omega) \\
& - 4''.040 \sin (4g + 2\omega) \\
& - 0''.338 \sin (5g + 2\omega) \\
& - 0''.027 \sin (6g + 2\omega) \\
& + 0''.007 \sin (2g + 4\omega) \\
& - 0''.092 \sin (3g + 4\omega) \\
& + 0''.400 \sin (4g + 4\omega) \\
& + 0''.092 \sin (5g + 4\omega) \\
& + 0''.013 \sin (6g + 4\omega) \\
& - 1''.198 \sin (2g' + 2\omega') \\
& - 0''.285 \sin (2g - 4g' + 2\omega - 4\omega) \\
& + s S_0.
\end{aligned}
$$

Coefficient of $n\delta z$.

[The comma points off six places of decimals.]

$$
\begin{aligned}
L_1 = & \ 1 \\
& + .109760,2 \cos g \\
& + .007526,9 \cos 2g \\
& + .000536,8 \cos 3g \\
& + .000038,9 \cos 4g \\
& + .000002,8 \cos 5g \\
& + .000221,2 \cos (g + 2\omega) \\
& - .003991,2 \cos (2g + 2\omega) \\
& - .000658,6 \cos (3g + 2\omega) \\
& - .000078,3 \cos (4g + 2\omega) \\
& - .000008,2 \cos (5g + 2\omega) \\
& - .000000,8 \cos (6g + 2\omega)
\end{aligned}
$$

$$
\begin{aligned}
&- .000001{,}3 \cos (3g + 4\omega) \\
&+ .000007{,}8 \cos (4g + 4\omega) \\
&+ .000002{,}2 \cos (5g + 4\omega) \\
&+ s\,S_1
\end{aligned}
$$

$$
\begin{aligned}
L_2 = &- .05488 \sin g \\
&- .00753 \sin 2g \\
&- .00080 \sin 3g \\
&- .00008 \sin 4g \\
&- .00011 \sin (g + 2\omega) \\
&+ .00399 \sin (2g + 2\omega) \\
&+ .00099 \sin (3g + 2\omega) \\
&+ .00016 \sin (4g + 2\omega) \\
&+ s\,S_2
\end{aligned}
$$

$$
\begin{aligned}
L_3 = &- .0183 \cos g \\
&- .0050 \cos 2g \\
&- .0008 \cos 3g \\
&+ .0027 \cos (2g + 2\omega) \\
&+ .0010 \cos (3g + 2\omega)
\end{aligned}
$$

The several parts of this expression for L are given in Table II, omitting the following terms, which are, however, all included in the column giving the concluded coefficients in L:—

1. The terms of L_0, explicitly given in the first of the preceding equations.
2. The expressions for $n\delta z$, $(n\delta z)^2 \times s S_2$, $(n\delta z)^3 \times R_{1,3}$, and $(n\delta z) \times R_{2,1}$.

The values of the last three expressions are as follows, the numbers within the parentheses being coefficients of g, g', ω, and ω', respectively:—

$n\delta z \times R_{2,1}$ = ″	$(n\delta z)^2 \times s S_2$ = ″	$(n\delta z)^3 \times R_{1,3}$ = ″
−.001 sin (3, 3, 2, 2)	−.002 sin (0, −2, 2, −2)	−.001 sin (2, 1, 2, 0)
+.001 sin (1, 2, 2, 2)	+.003 sin (2, −2, 2, −2)	+.001 sin (2, −1, 2, 0)
−.005 sin (2, 2, 2, 2)	+.002 sin (3, −2, 2, −2)	−.002 sin (0, 2, 0, 2)
−.020 sin (3, 2, 2, 2)	+.002 sin (−1, 2, 0, 2)	−.004 sin (1, 2, 0, 2)
−.005 sin (4, 2, 2, 2)	+.004 sin (0, 2, 0, 2)	−.002 sin (2, 2, 0, 2)
−.002 sin (4, 1, 4, 0)	−.002 sin (2, −2, 4, −2)	+.002 sin (2, −2, 4, −2)
+.002 sin (4, −1, 4, 0)	−.002 sin (3, −2, 4, −2)	+.003 sin (3, −2, 4, −2)
+.001 sin (5, −1, 4, 0)	−.002 sin (4, −6, 6, −6)	+.003 sin (4, −2, 4, −2)
−.003 sin (4, −2, 6, −2)	−.002 sin (5, −6, 6, −6)	+.002 sin (5, −2, 4, −2)
+.016 sin (5, −2, 6, −2)	−.002 sin (2, −6, 4, −6)	−.001 sin (1, −6, 4, −6)
+.013 sin (6, −2, 6, −2)	−.002 sin (3, −6, 4, −6)	−.001 sin (2, −6, 4, −6)
+.002 sin (7, −2, 6, −2)		−.001 sin (6, −6, 8, −6)
		−.001 sin (7, −6, 8, −6)

In Table II the column "Sum" contains the sums of the terms actually given in the preceding columns of the table.

The next column gives the complete coefficient of each term in the ecliptic longitude, and is formed by adding to the column "Sum" the omitted terms just referred to.

The last column gives, for the larger terms, the elements which they principally contain as factors. If these elements be changed, the coefficients must be changed by corresponding quantities.

§ 2.

REDUCTION OF THE PRECEDING EXPRESSIONS TO UNIFORM ELEMENTS, AND COMPARISON WITH DELAUNAY.

The coefficients of the preceding inequalities contain as factors certain elements for which different investigators adopt different values. It is essential to a clear presentation of results that they should be reduced to a uniform and well-defined set of elements having given values We therefore commence by reducing the theories of both HANSEN and DELAUNAY to such a system. The elements principally referred to are—

(α) The ratio of the mean motions of the sun and moon.
(β) The lunar eccentricity.
(γ) The solar parallax.
(δ) The solar eccentricity.
(ϵ) The inclination of the moon's orbit.

Really, all these elements are contained in all the inequalities in a very complex manner. But there is so little doubt about their true numerical values that it is only necessary to take account of their changes when they appear as factors in coefficients of considerable magnitude. The extent to which each term is affected can be roughly seen from its analytic expression given by DELAUNAY at the end of his *Theorie du Mouvement de la Lune*, Tome II. We take up the several elements in order.

(α) *Ratio of mean motions.* This element is so certain that no reduction need be made on account of it. It is true that theoretical motions of the lunar node and perigee must implicitly enter in connection with this element. But, from a rough examination of HANSEN's integration coefficients on pp. 350–352 of his *Darlegung*, I do not think any of the larger coefficients will be affected by as much as $\frac{1}{100000}$ of their entire amount by any admissible change of these motions.

(β) *Eccentricity of moon's orbit.* The eccentricities used by the two investigators are not directly comparable, but may be most conveniently compared by reducing each to the coefficient of g in the expression for the moon's ecliptic longitude. DELAUNAY uses AIRY's value, given in his last paper on the elements of the moon's orbit.* HANSEN corrected his eccentricity for use in his tables, as already mentioned. The writer obtained a small but well-marked correction to HANSEN's value from the Green-

* Memoirs Royal Astronomical Society, Vol. XXIX.

wich observations 1846–'74, and the Washington observations 1862–'74. The four values of the coefficient in question are:—

AIRY, used by DELAUNAY, ...	22639″.06
HANSEN, used in Theory, ...	22637″.15
HANSEN, used in Tables, ...	22640″.15
Corrected value found in 1876,* . .	22639″.58

Although there is no reasonable doubt that the eccentricity of HANSEN's tables requires a negative correction, it will be adopted for the purposes of comparison because it is now the standard of the ephemerides with which subsequent comparisons must be made. All the terms having e as a coefficient, must therefore be increased by the factor

$$\frac{.00000728}{.05490} = .0001326,$$

and those having e^2 by double this factor. The coefficients in e must, in DELAUNAY's theory, be increased by the factor

$$\frac{1''.09}{22639''} = .0000482.$$

(γ) *Solar parallax.* HANSEN's theory does not set out with a definite solar parallax, but with a ratio of the mean distances of the sun and moon, which ratio again is not the usual one, because HANSEN's a and a' are the same functions of the motion of mean anomaly that the usual a and a' are of the sidereal motions. We must therefore adopt an indirect process for finding the relation of solar parallax and parallactic equation on his theory. He finds that his theoretical coefficient has to be multiplied by the factor 1.03573 to make it agree with observation: and then, in § 266 of his *Darlegung*, he deduces the solar parallax 8″.9159. Dividing this parallax by the preceding factor, we conclude that the parallax of his theory is:—

$$8''.6085.$$

In turning his theory into numbers DELAUNAY used 8″.75. The parallax to which both theories will be actually reduced is:—

$$8''.848.$$

Hence, HANSEN's terms having the parallax as a factor must be increased by the factor

$$.02785,$$

and DELAUNAY's by the factor

$$.01120.$$

(δ) *The solar eccentricity.* The solar eccentricity of HANSEN's theory is:—

$$e' = 0.01679226 \qquad \text{(Epoch 1800).}$$

* Papers published by the Commission on the Transit of Venus. Part III.

DELAUNAY uses LE VERRIER'S value:—

$$e' = 0.01677106 \qquad \text{(Epoch 1850).}$$

In strictness these two values are not comparable, owing to the different form of HANSEN'S solar theory; but since HANSEN neglects perturbations of the earth's motion in his lunar theory, it may be assumed that there will be no difference between the form in which the eccentricity enters into the two theories. If we carry LE VERRIER'S eccentricity back to 1800 with his secular variation, we shall have:—

$$e' = 0.01679228 \qquad \text{(Epoch 1800).}$$

This may be regarded as absolutely identical with HANSEN'S value for the same epoch. So, adopting 1800 as the epoch, we have only to increase DELAUNAY'S coefficients in e' by the factor

$$\frac{.00002122}{.01677} = .001265.$$

Or, we may reduce HANSEN'S values to 1850 by dividing them by 1.001265, when they will be comparable with DELAUNAY'S.

The theories of HANSEN and DELAUNAY, thus reduced to a uniform and consistent set of elements, are given and compared in Table III. DELAUNAY'S results are frequently doubtful by a small fraction of a second, owing to the slow convergence of the series in powers of m, and the table has been arranged so as to show the extent of the uncertainty thus arising.

Following the indices expressing the arguments are given, first, HANSEN'S coefficients formed from the values in Table II by multiplying by the appropriate factors for reduction already given. They are only given to $0''.01$, but should the thousandth of seconds be required they are readily obtainable.

The corresponding coefficients of DELAUNAY are derived principally from his presentation of numerical results in the additions to the *Connaissance des Temps* for 1869. On pages 11 to 21 of that paper are given the sums of the terms in each coefficient which were actually computed by him. The parallactic terms, as given by DELAUNAY, are still to be multiplied by $\frac{1 - \lambda}{1 + \lambda}$, λ being the ratio of the mass of the moon to that of the earth. Putting, with HANSEN, $\lambda = \frac{1}{80}$, the coefficient will be $\frac{79}{81}$. The sums, corrected for this coefficient and for difference of elements, are given in the column *Delaunay* (1). Had all the appreciable terms been actually computed, these coefficients would have been the definitive ones of DELAUNAY'S theory. But it was frequently found that the terms, even of the ninth order, where the development ceased, were still appreciable; it was, therefore, necessary to estimate the probable sum of the omitted terms of higher orders from the law of the series as observed in the terms actually computed. These estimates can have no true mathematical foundation, because there is no proof of the actual law of the series.* Still, there is a high degree of probability in favor of each one being at least a rude approximation to the truth. A rigorous computation would probably show that a majority differed less than $\frac{1}{4}$ of their amount from the true values, though here and there one might be found entirely illusory. The coefficients of longitude, modified by these estimated additions, are given by DELAUNAY on pages 38–40 of the paper referred to, and are reproduced, with the necessary corrections for changes of elements, in the column *Delaunay* (2).

The difference of these results, given in the next column, is the correction apparently applied by DELAUNAY for the uncomputed terms. It will be noted that we have no independent statement of these terms to refer to, and can only infer their values from the differences between the printed results (1) and (2)

Finally, we have the difference, *Hansen* minus *Delaunay* (2) showing the discrepancies still outstanding between the two theories Each one can judge for himself how far these discrepancies arise from the uncertainty of DELAUNAY'S semi-empirical corrections, and how far from errors in the two theories.

. One or two terms are worthy of a special examination, and among these the parallactic equation takes the first rank, as upon it depends the value of the solar parallax to be derived from a given observed value of this equation Arranging DELAUNAY'S terms according to the power of m, which enters as a factor, the result will be that given below under the head P_1. DELAUNAY omits terms in γ^2 after m^3, and terms in e^2 after m^5. Correcting the result for an estimated value of these terms, derived by induction, we shall have those given under the head P_2. It will be seen that the terms follow a nearly regular law up to m^6, but that m^7 deviates from this law. Assuming this term to be in error, and estimating the value of it and the higher terms as those of a geometrical progression with the ratio $\frac{4}{10}$ we have the results P_3.

	P_1	P_2	P_3
Terms in m	− 73″.1760	− 73″.18	− 73″.18
m^2	− 34 .3021	− 34 .30	- 34 .30
m^3	− 12 .0082	− 12 .01	− 12 .01
m^4	− 4 .6812	− 4 .50	− 4 .50
m^5	− 1 .9815	− 1 .89	− 1 .89
m^6	− 0 .7122	− 0 .72	− 0 .72
m^7	− 0 .3811	− 0 .38	− 0 .48
Sum	− 127″.2423	− 126″.98	− 127″.08

Our choice must lie between the results P_2 and P_3. If we adopt the former we may add 0″.26 as an estimate of omitting terms giving :—

$$P = -127''.24; \quad P' = \frac{79}{81} P = -124''.10.$$

* It may be remarked that in the series for the secular acceleration DELAUNAY found the terms of a higher order actually to change their sign, directly contrary to the estimate which would have been formed from those of a lower order.

If we adopt the latter we have

$$P = -127''.08; \qquad P' = \frac{79}{81} P = -123''.94.$$

Multiplying by the coefficient 1.0112 to reduce to the parallax 8''.848 the result will be:—

$$(2) \qquad -125''.49$$
$$(3) \qquad -125''.33.$$

HANSEN'S coefficient, $-125''.43$, falls between these results and may be regarded as certainly correct within less than 0''.1.

The other term referred to is that depending on the argument:—

$$g - g' + 2\omega - 2\omega',$$

of which the principal parts of the coefficient are, in DELAUNAY'S theory,—

3d order, . . .	$-53''.10$
4th order, . . .	$-5''.80$
5th order, . . .	$+10''.15$
6th order, . . .	$+9''.34$
7th order, . . .	$+5''.62$
8th order, . . .	$+2''.90$
9th order, . . .	$+1''.43$

DELAUNAY seems to have taken 1''.18 as the probable sum of the omitted terms, whereas they should have been taken as 0''.94 to agree with HANSEN.

§ 3.

LATITUDE.

Taking HANSEN'S expression for the moon's latitude:—

$$\sin\beta = \sin I \sin(f + \omega) + s;$$

the first step is to form the expression $\sin(f + \omega)$ in terms of g, ω, etc. This may be done in two ways. By the first we express the required quantity as a function of z, and then put $g + n\delta z$ for g and develop in powers of $n\delta z$. By the theory of elliptic motion the expression of $\sin(f + \omega)$ in terms of z will be

$$
\begin{aligned}
\sin(f+\omega) = \quad & - \frac{625}{9216} e^6 \sin(-5z+\omega) \\
& - \frac{1}{15} e^5 \sin(-4z+\omega) \\
& + \left(- \frac{9}{128} e^4 + \frac{9}{320} e^6 \right) \sin(-3z+\omega) \\
& + \left(- \frac{1}{12} e^3 + \frac{1}{48} e^5 \right) \sin(-2z+\omega) \\
& + \left(- \frac{1}{8} e^2 + \frac{1}{48} e^4 + \frac{37}{3072} e^6 \right) \sin(-z+\omega) \\
& - e \sin\omega \\
& + \left(1 - e^2 + \frac{7}{64} e^4 - \frac{5}{288} e^6 \right) \sin(z+\omega) \\
& + \left(e - \frac{5}{4} e^3 + \frac{17}{48} e^5 \right) \sin(2z+\omega) \\
& + \left(\frac{9}{8} e^2 - \frac{27}{16} e^4 + \frac{765}{1024} e^6 \right) \sin(3z+\omega) \\
& + \left(\frac{4}{3} e^3 - \frac{7}{3} e^5 \right) \sin(4z+\omega) \\
& + \left(\frac{625}{384} e^4 - \frac{625}{192} e^6 \right) \sin(5z+\omega) \\
& + \frac{81}{40} e^5 \sin(6z+\omega) \\
& + \frac{117649}{46080} e^6 \sin(7z+\omega).
\end{aligned}
$$

If we now substitute for z, $g + n\delta z$, for e its numerical value, and develop, putting:—

$$\sin I \sin(f+\omega) = F_0 + F_1 n\delta z + F_2 (n\delta z)^2 + F_3 (n\delta z)^3$$

we shall have

$$
\begin{aligned}
F_0 \text{ (in arc)} = & - \quad 0''.012 \sin(-3g+\omega) \\
& - \quad 0''.255 \sin(-2g+\omega) \\
& - \quad 6''.968 \sin(-g+\omega) \\
& - 1015''.834 \sin\omega \\
& + 18447''.342 \sin(g+\omega) \\
& + 1012''.011 \sin(2g+\omega) \\
& + \quad 62''.458 \sin(3g+\omega) \\
& + \quad 4''.061 \sin(4g+\omega)
\end{aligned}
$$

$$
\begin{aligned}
F_0 \text{ (in arc) (cont'd)} = &+ 0''.272 \sin (5g + \omega)\\
&+ 0''.019 \sin (6g + \omega)\\
&+ 0''.001 \sin (7g + \omega)
\end{aligned}
$$

$$
\begin{aligned}
F_0 \text{ (in radius)} = &- .000\,0001 \sin (-3g + \omega)\\
&- .000\,0012 \sin (-2g + \omega)\\
&- .000\,0338 \sin (-g + \omega)\\
&- .004\,9249 \sin \omega\\
&+ .089\,4352 \sin (g + \omega)\\
&+ .004\,9064 \sin (2g + \omega)\\
&+ .000\,3028 \sin (3g + \omega)\\
&+ .000\,0197 \sin (4g + \omega)\\
&+ .000\,0013 \sin (5g + \omega)\\
&+ .000\,0001 \sin (6g + \omega)
\end{aligned}
$$

$$
\begin{aligned}
F_1 = &+ .000\,0002 \cos (-3g - \omega)\\
&+ .000\,0025 \cos (-2g - \omega)\\
&+ .000\,0338 \cos (-g - \omega)\\
&+ .089\,4352 \cos (g - \omega)\\
&+ .009\,8127 \cos (2g - \omega)\\
&+ .000\,9084 \cos (3g - \omega)\\
&+ .000\,0788 \cos (4g - \omega)\\
&+ .000\,0066 \cos (5g - \omega)\\
&+ .000\,0005 \cos (6g - \omega)
\end{aligned}
$$

$$
\begin{aligned}
F_2 = &+ .000\,02 \sin (-g + \omega)\\
&- .044\,72 \sin (g + \omega)\\
&- .009\,81 \sin (2g + \omega)\\
&- .001\,36 \sin (3g + \omega)\\
&- .000\,16 \sin (4g + \omega)\\
&- .000\,02 \sin (5g + \omega)
\end{aligned}
$$

$$
\begin{aligned}
F_3 = &- .0149 \cos (g + \omega)\\
&- .0065 \cos (2g + \omega)\\
&- .0014 \cos (3g + \omega)
\end{aligned}
$$

As a check upon the value of sin I sin $(f + \omega)$ a second method of computing it was adopted, as follows. Let us put:—

$$\delta f = f - g.$$

Then

$$
\begin{aligned}
\sin (f + \omega) &= \sin (g + \omega + \delta f)\\
&= \cos \delta f \sin (g + \omega)\\
&\quad + \sin \delta f \cos (g + \omega).
\end{aligned}
$$

From the numerical value of δf already given the powers of this quantity were formed, and thence its cosine and sine from the formulæ:—

$$\cos \delta f = 1 - \frac{\delta f^2}{1.2} + \text{etc.}$$

$$\sin \delta f = \delta f - \frac{\delta f^3}{1.2.3} + \text{etc.}$$

These expressions were then multiplied by the sine and cosine of $(g + \omega)$.

The mean difference between the coefficients in sin I sin $(f + \omega)$ found by the two methods was less than 0″.003, the largest one being ″.010.

Adding HANSEN's s to this expression we have the value of sin β. Then β itself is obtained by the formula

$$\beta = \sin \beta + \frac{1}{6} \sin^3 \beta + \frac{3}{40} \sin^5 \beta.$$

The principal parts of β are given in Table IV, of which the columns referring to HANSEN's theory seem to need no explanation.

§ 4.

REDUCTION OF THE LATITUDE AND COMPARISON WITH DELAUNAY.

All the terms of the latitude contain the inclination of the moon's orbit as a factor, and are therefore to be multiplied by such a constant coefficient that the principal term of the latitude shall agree with observation. The transformed expressions of HANSEN, given in Table IV, lead to a consistent theory in which the coefficient of the principal term of the latitude is 18463″.248. The expressions of DELAUNAY also lead to a theory, in which this coefficient is 18461″.26. Each of these is to be multiplied by such a factor as shall reduce it to the value implicitly adopted in HANSEN's tables. There HANSEN adopts:—

$$I = 5^\circ\ 8'\ 39''.96,$$

which is less by 8″.04 than that of the theory. Hence, from this alone would follow the correction:—

$$-8''.04 \sin (f + \omega).$$

But, the tables contain, among the perturbations, two terms which depend mainly on the same argument, namely:—

$$2''.705 \sin (f + \omega),$$

which, developed by putting $g + 2e \sin g$ for f, appears as a perturbation, and

$$3''.70 \sin (g + \omega),$$

which is attributed to the separation of the centers of figure and of gravity of the moon. The sum of the first two expressions being developed, become

$$\begin{aligned} &- 5''.319 \sin (g + \omega) \\ &+ 0''.293 \sin \omega \\ &- 0''.292 \sin (2g + \omega). \end{aligned}$$

Adding the third, the term in $g + \omega$ will become

$$- 1''.619 \sin (g + \omega).$$

We are not concerned with the terms in ω and $2g + \omega$. The greater part of their amount may be considered as a *quasi* perturbation, due to the figure of the moon, and implicitly contained in the tables, but not belonging to the problem of three bodies.

With the last correction the term in $g + \omega$ becomes

$$18461''.629 \sin (g + \omega).$$

which is the coefficient implicitly contained in HANSEN's tables.

To this the writer found a correction of $-0''.15$ from Greenwich and Washington observations 1862–'74, but it will be retained without change. Hence all the coefficients in HANSEN's β, as given in Table IV, are to be diminished by the factor

$$.000088.$$

and those of DELAUNAY are to be increased by the factor

$$.000020.$$

The terms in e and e' are to be modified by the same coefficients as in the case of the longitude. The only terms which will be appreciably affected by the change of e are those depending on ω and $2g + \omega$.

The modifications here indicated have not been made in the results, because they are so slight, and affect so few terms, that each one can make them for himself.

The column *Delaunay* (1) contains, as before, the sum of the terms actually computed by DELAUNAY, and given by him in the *Connaissances des Temps* for 1869.

In column *Delaunay* (2) his coefficients are corrected by the higher terms, of which the value has been estimated by induction. DELAUNAY himself did not give these additions, so that they had to be estimated by the writer.

§ 5.

PARALLAX.

HANSEN's theory gives the perturbations of the natural logarithm of the moon's radius vector, which are the negative of the perturbations of the logarithm sine parallax. The value of w, in seconds of arc, is found in the *Darlegung*, Part I, pages 409–411, and Part II, pages 224–226, 258, and 268. The moon's parallax p is given by HANSEN under the form

$$\log \sin p = \log \frac{D\,(1 + e \cos f)}{a\,(1 - e^2)} - w,$$

in which D is the radius of the earth at the latitude of which the sine is $\sqrt{\frac{1}{3}}$, and a the moon's mean distance in the HANSENIAN theory, which is different in definition from the mean distance of the ordinary theories. It is not, however, necessary to reduce the one to the other directly, because they may be most satisfactorily compared by the values of the constant of parallax to which they lead.

Changing the logarithms to natural quantities and developing in powers of w, the above expression gives:—

$$\sin p = \frac{D}{a} \cdot \frac{1 + e \cos f}{1 - e^2} \left(1 - w + \frac{w^2}{2} - \text{etc.}\right)$$

and then

$$p = \sin p + \frac{\sin^3 p}{6} + \text{etc.}$$

In developing $e \cos f$ two methods of computation were used, as in the computation of the principal term of the latitude.

1. From CAYLEY's tables we have

$$\begin{aligned} \cos f = -e &+ \left(1 - \frac{9}{8}e^2 + \frac{25}{192}e^4\right) \cos z \\ &+ \left(e - \frac{4}{3}e^3\right) \cos 2z \\ &+ \left(\frac{9}{8}e^2 - \frac{225}{128}e^4\right) \cos 3z \\ &+ \frac{4}{3}e^3 \cos 4z \\ &+ \frac{625}{384}e^4 \cos 5z \end{aligned}$$

and then by substituting $g + n\delta z$ for z we have $\cos f$ developed in multiples of g, etc.

2. Putting

$$f = g + \delta f$$

we have

$$\cos f = \cos \delta f \cos g - \sin \delta f \sin g.$$

The value of $\frac{D}{a}$ was derived by HANSEN from the length of the seconds pendulum and the dimensions of the earth as found by BESSEL. The derivation is given in the *Astronomische Nachrichten*, Volume XVII, page 300. The data made use of are:—

D, radius of earth under the parallel arc $\sin \sqrt{\frac{1}{3}}$ . .	6370063 metres
P, length of seconds pendulum under same parallel .	$0^m.992666$
m, mass of the moon	$\frac{1}{80}$.

The result is

$$\log \frac{D}{a} = 8.2170139.$$

He gives as the resulting constant part of the sine of the parallax

$$3422''.06,$$

and the changes in the constant produced by small changes in the data:—

Increase of $0^{mm}.1$ in P varies the constant by	$-0''.11$
Increase of 1000^{m} in D varies the constant by . . .	$+0''.18$
Increase of unity in denominator of m	$+0''.17$

The development subsequently given leads to a constant of

$$3422''.09,$$

a result $0''.03$ greater than that stated by HANSEN.

In comparing the parallaxes of HANSEN and DELAUNAY the only element which will materially affect the result is the constant of parallax: a comparison of the different values of this constant, which have been recently obtained, will therefore be of interest. Three distinct methods of obtaining this important element have been applied

(α). The theoretical method founded on KEPLER'S third law as expressed in the theory of gravitation, and derived fundamentally from the equation

$$a^3 n^2 = m + M,$$

a being the mean distance of the moon, which is immediately connected with the parallax; n the mean motion, of the value of which there is no doubt, and m and M the masses of the moon and earth, expressed in appropriate units, the determination of which is the most doubtful part of the problem.

(β). Measures of the moon's position made at two distant stations, and reduced to a common moment.

(γ). Meridian declinations of the moon made at the same station, and reduced on the hypothesis that the undisturbed geocentric orbit is a great circle.

The last method is not well adapted to give a certain result, owing to the constant errors with which measures of absolute declinations are affected. We shall therefore confine our consideration to the first two.

Two determinations by method (α), that of HANSEN, just quoted, and that of ADAMS in the *Monthly Notices*, Vol. XIII, and the British *Nautical Almanac* for 1856, are available.

The data used by Mr. ADAMS are:—

D, from BESSEL, and therefore the same as HANSEN.

P,* 3.256 89 English feet, or . . . $0^{m}.992712$.

m, mass of moon, ... $\frac{1}{81.5}$.

* This value in English feet was kindly communicated by Mr. ADAMS himself, not being explicitly quoted in his published paper.

The resulting value of the constant of the sine is given as 3422″.325. To compare it with HANSEN we have:—

$$\begin{array}{llll} \text{Change in } D = 0, & \text{- - -} & \text{change of } \pi_0 = 0 \\ \text{"} \quad \text{"} \quad P = +0^{m\,m}.046, & \text{"} \quad \text{"} & -0''.05 \\ \text{"} \quad \text{"} \quad \frac{1}{m} = +1.5, & \text{"} \quad \text{"} & +0''.26 \end{array}$$

Applying the correction +0″.21 to HANSEN's constant, the result would be either 3422″.27 or 3422″.30, according as we accept HANSEN's original constant or that deduced from the data of his lunar tables. The latter is probably the value to be preferred.

If we reduce the values both of HANSEN and ADAMS to HANSEN's data, according to the system already adopted, the results will be:—

Constant of sine parallax,	HANSEN,	3422″.09.
" " "	ADAMS,	3422″.12.
Constant of parallax itself,	HANSEN,	3422″.25.
" " "	ADAMS,	3422″.28.

The constant of reduction from the sine to the parallax itself is + 0″.157.

β. The most recent determinations of the moon's parallax by measurement are those of Mr. BREEN (Memoirs R. A. S. XXXII) and of Mr. STONE (Ibid. XXXIV). Both are founded on Cape observations and both lead to a constant of

3422″.70.

It is not distinctly stated whether this is the constant of the parallax itself or of its sine. Mr. BREEN's introduction (l. c. pp. 116, 117) seems to imply that he used Mr. ADAMS's expression for sine parallax as the parallax itself in reducing the Cape observations. But, in the reduction of the Greenwich observations, he applies ADAMS's correction to the parallax of AIRY's lunar reductions, which gives the parallax itself. To put the matter into another shape: On p. 116 Mr. BREEN has 3422″.32 as the constant of parallax. On p. 132 he has a constant correction of 0″.68 to the AIRY-PLANA parallax, of which the constant is 3421″.80, which gives 3422″.48 as the constant of parallax.

We shall probably make a near approximation to the truth by assuming that Mr. BREEN's mean provisional constant was 3422″.40, and as he deduced a correction of +0″.38 this would give us his result:—

Constant of parallax, - - -	3422″.78
Constant of sine, - - - -	3422″.62

Mr. STONE also finds a correction of +0″.38 to Mr. ADAMS's parallax. This would give:—

Constant of parallax, - - -	3422″.86
Constant of sine, - - - -	3422″.70

The evidence is therefore in favor of a positive correction to HANSEN's constant; but, in accordance with the practice in other parts of this paper, the results as printed are all founded on HANSEN's fundamental data.

In the Table V the columns contain—

(1). The value of $\frac{D}{a}\cdot\frac{1}{1-e^2}(1+e\cos f)$, expressed in seconds of arc.

(2). The product of this quantity by $-w+\frac{w^2}{2}$.

(3). The coefficients for HANSEN's sine parallax, formed by adding (1) and (2).

If the parallax itself is required, it may be found by adding the reduction from the sine to the parallax itself, namely:—

$$+0''.157 \quad +0''.025\cos g$$
$$+0''.004\cos(g-2g^1+2\omega-2\omega^1)$$
$$+0''.004\cos(2g-2g^1+2\omega-2\omega^1).$$

(4). The coefficients of DELAUNAY's sine parallax, so far as actually computed by him. As he stopped at the terms of the fifth order, the hundredths of seconds are not always definitive.

(5). The same, with the addition of quantities estimated by induction to represent the omitted terms of higher orders.

(6) The corrections applied in the preceding column to obtain the most probable values of the coefficients.

(7) The deviation of HANSEN's coefficients from the second set of DELAUNAY's.

As some of DELAUNAY's terms are doubtful from the insufficient convergence of his series, the coefficients of ADAMS's parallax, found in the *Monthly Notices R. A. S.*, Vol. XIII, p. 263, have been added for comparison. It will be seen that they agree closely with the coefficients of HANSEN, though derived independently of them.

Table I.—*Value of* $n\delta z$, *from* Hansen, *together with its powers.*

g	g'		$n\delta z$ sin.	$(n\delta z)^2$ cos.	$(n\delta z)^3$ sin	$(n\delta z)^4$ cos.
			″	″	″	″
0	0		0.000	+60.860	0.000	+ 0.035
1	0		0.000	+16.934	− 0.008	+ 0.043
2	0	−	4.604	+ 0.809	− 0.004	+ 0.009
3	0	−	0.176	+ 0.015	− 0.002	. .
4	0	−	0.009	+ 0.002	. .	. .
−3	−1	+	0.029	− 0.003	. .	. .
−2	−1	+	1.097	− 0.005	+ 0.029	. .
−1	−1	+	73.234	+ 1.557	+ 0.288	+ 0.002
0	−1	+	657.468	+ 5.093	+ 0.638	+ 0.008
1	−1	+	111.681	+ 3.477	+ 0.320	+ 0.004
2	−1	+	1.215	+ 0.125	+ 0.042	+ 0.001
3	−1	+	0.026	+ 0.005	+ 0.001	. .
−2	−2	+	0.002	− 0.017	+ 0.001	. .
−1	−2	+	0.800	− 0.185	+ 0.014	. .
0	−2	+	7.319	− 0.953	+ 0.034	. .
1	−2	+	2.159	− 0.233	+ 0.023	. .
2	−2	+	0.035	− 0.030	+ 0.003	. .
3	−2		. .	− 0.001	. .	. .
−1	−3	+	0.011	− 0.005	. .	. .
0	−3	+	0.075	− 0.019	. .	. .
1	−3	+	0.044	− 0.007	. .	. .
$2\omega - 2\omega'$						
−1	0		. .	+ 0.003	− 0.001	. .
0	0	−	0.094	+ 0.043	− 0.010	. .
1	0	−	2.524	+ 0.082	− 0.039	. .
2	0	−	0.052	+ 0.010	− 0.024	. .
3	0		. .	− 0.002	− 0.004	. .
−1	−1	−	0.040	. .	+ 0.001	. .
0	−1	+	3.665	+ 2.324	+ 0.046	+ 0.007
1	−1	−	27.620	+15.444	+ 0.085	+ 0.015
2	−1	−	23.006	+ 8.499	+ 0.045	+ 0.012
3	−1	−	1.337	+ 0.984	+ 0.007	+ 0.003
4	−1	−	0.060	+ 0.041	. .	. .
−2	−2	−	0.026	− 0.003	. .	. .
−1	−2	−	1.893	+ 0.085	+ 0.007	. .
0	−2	−	41.648	+ 0.305	+ 0.762	. .
1	−2	+	4466.992	+ 1.867	+ 2.381	+ 0.002
2	−2	+	2144.995	+ 1.494	+ 1.750	. .
3	−2	+	60.020	+ 0.420	+ 0.414	. .
4	−2	+	2.053	− 0.071	+ 0.018	. .
5	−2	+	0.084	+ 0.004	. .	. .
6	−2	+	0.004	. .	. .	. .
−1	−3	−	0.082	− 0.006	. .	. .
0	−3	−	2.352	− 1.437	+ 0.061	− 0.005
1	−3	+	198.103	−14.857	+ 0.216	− 0.015
2	−3	+	155.047	− 9.149	+ 0.202	− 0.011
3	−3	+	5.166	− 1.341	+ 0.059	− 0.003
4	−3	+	0.198	− 0.047	+ 0.003	. .
5	−3	+	0.009	− 0.001	. .	. .

g	g'		$n\delta z$ sin.	$(n\delta z)^2$ cos.	$(n\delta z)^3$ sin.	$(n\delta z)^4$ cos.
$2\omega - 2\omega'$			″	″	″	″
0	−4	−	0.109	− 0.077	− 0.004	. .
1	−4	+	7.035	− 0.846	− 0.028	. .
2	−4	+	7.738	− 0.718	− 0.014	. .
3	−4	+	0.257	− 0.122	− 0.003	. .
4	−4	+	0.011	− 0.006	. .	. .
0	−5		. .	− 0.003	. .	. .
1	−5	+	0.240	− 0.035	− 0.002	. .
2	−5	+	0.329	− 0.037	− 0.002	. .
3	−5	+	0.012	− 0.007	. .	. .
2ω						
1	2		. .	− 0.003	. .	. .
2	2	+	0.002	0.000	. .	. .
−1	1		. .	+ 0.003	. .	. .
0	1	+	0.037	− 0.039	− 0.001	. .
1	1	−	0.351	− 0.210	− 0.012	. .
2	1	+	0.127	+ 0.012	− 0.006	. .
3	1	+	0.001	+ 0.003	. .	. .
−1	0	−	0.070	. .	. .	. .
0	0	+	5.846	− 0.266	− 0.024	. .
1	0	−	85.224	+ 1.033	− 0.072	. .
2	0	+	4.303	+ 0.941	− 0.025	. .
3	0	+	0.094	+ 0.073	+ 0.001	. .
4	0	+	0.001	+ 0.003	. .	. .
0	−1	+	0.046	+ 0.004	− 0.001	. .
1	−1	+	0.279	+ 0.295	+ 0.005	. .
2	−1	+	0.119	+ 0.077	+ 0.005	. .
3	−1	+	0.003	+ 0.053	+ 0.001	. .
1	−2		. .	+ 0.003	. .	. .
2	−2	+	0.004	+ 0.002	. .	. .
$2\omega'$						
−1	4		. .	− 0.004	. .	. .
0	4	−	0.114	− 0.017	. .	. .
−3	3		. .	− 0.002	.	. .
−2	3	+	0.012	+ 0.010	. .	. .
−1	3	+	0.604	− 0.069	+ 0.002	. .
0	3	−	3.449	− 0.346	+ 0.004	. .
1	3	−	0.152	− 0.036	− 0.001	. .
−3	2	+	0.005	+ 0.002	. .	. .
−2	2	+	0.162	+ 0.038	+ 0.003	. .
−1	2	+	10.038	− 0.756	− 0.021	. .
0	2	−	81.905	− 1.779	− 0.071	. .
1	2	−	4.722	+ 0.116	− 0.022	. .
2	2	−	0.075	. .	− 0.002	. .
−2	1	−	0.006	− 0.005	. .	. .
−1	1	−	0.105	+ 0.006	− 0.004	. .
0	1	+	2.064	+ 0.274	− 0.007	. .
1	1	−	0.043	+ 0.058	− 0.002	. .
2	1		. .	+ 0.003	. .	. .
0	0	+	0.012	− 0.002	. .	. .
0	−1		. .	. .	+ 0.003	. .

TABLE I.—*Value of nδz, &c.*—Continued.

g	g′	nδz sin.	$(n\delta z)^2$ cos.	$(n\delta z)^3$ sin.	$(n\delta z)^4$ cos.
2ω + 2ω′		″	″	″	″
0	2	. .	+ 0.007	. .	. .
1	2	− 0.014	− 0.033	. .	. .
2	2	+ 0.006	. .	. .	. .
ω − ω′					
0	1	+ 0.007	+ 0.010	. .	. .
1	1	− 0.031	+ 0.052	+ 0.001	. .
2	1	. .	+ 0.007	. .	. .
−1	0	+ 0.290	− 0.013	+ 0.002	. .
0	0	+ 0.316	− 0.069	+ 0.017	. .
1	0	+ 17.566	− 0.382	+ 0.019	. .
2	0	+ 0.259	− 0.044	+ 0.005	. .
−1	−1	− 0.564	− 0.021	− 0.005	− 0.001
0	−1	− 11.400	− 2.709	− 0.052	− 0.002
1	−1	− 121.335	− 1.631	− 0.113	− 0.002
2	−1	− 1.610	− 0.163	− 0.042	. .
3	−1	− 0.037	− 0.012	− 0.001	. .
−1	−2	− 0.009	+ 0.006	− 0.001	. .
0	−2	− 0.147	+ 0.343	− 0.016	. .
1	−2	− 0.562	+ 0.476	− 0.014	. .
2	−2	− 0.081	+ 0.074	− 0.004	. .
3	−2	− 0.006	+ 0.004	. .	. .
0	−3	− 0.007	+ 0.012	+ 0.001	. .
1	−3	+ 0.041	+ 0.015	+ 0.002	. .
3ω − 3ω					
2	−1	. .	+ 0.003	+ 0.001	. .
3	−1	. .	+ 0.002	+ 0.001	. .
0	−2	. .	− 0.007	. .	. .
1	−2	− 0.038	− 0.011	− 0.003	. .
2	−2	+ 0.272	− 0.410	− 0.014	. .
3	−2	+ 0.123	− 0.201	− 0.007	. .
4	−2	+ 0.009	− 0.009	. .	. .
0	−3	− 0.002	+ 0.005	+ 0.001	. .
1	−3	− 1.092	+ 0.210	− 0.044	. .
2	−3	− 3.154	+ 2.708	− 0.049	+ 0.003
3	−3	+ 0.621	+ 1.279	− 0.017	+ 0.001
4	−3	+ 0.018	+ 0.053	− 0.002	. .
5	−3	+ 0.001	. .	. .	. .
1	−4	− 0.066	+ 0.018	+ 0.005	. .
2	−4	− 0.229	+ 0.150	+ 0.011	. .
3	−4	+ 0.078	+ 0.100	+ 0.009	. .
4	−4	+ 0.003	+ 0.006	+ 0.001	. .
2	−5	− 0.012	+ 0.004	. .	. .
3	−5	+ 0.007	+ 0.004	. .	. .
ω + ω′					
1	2	. .	+ 0.007	. .	. .
−1	1	. .	− 0.001	. .	. .
0	1	+ 0.050	+ 0.045	+ 0.002	. .
1	1	+ 0.757	− 0.047	+ 0.002	. .
2	1	. .	− 0.004	. .	. .

g	g′	nδz sin.	$(n\delta z)^2$ cos.	$(n\delta z)^3$ sin.	$(n\delta z)^4$ cos.
ω + ω′		″	″	″	″
0	0	+ 0.040	− 0.007	. .	. .
1	0	+ 0.024	− 0.001	. .	. .
3ω − ω′					
2	0	.	+ 0.007	. .	. .
1	−1	. .	− 0.002	. .	. .
2	−1	+ 0.037	− 0.069	− 0.001	. .
3	−1	+ 0.010	− 0.007	. .	. .
1	−2	. .	− 0.001	. .	. .
2	−2	. .	− 0.002	. .	. .
ω − 3ω′					
1	−2	. .	− 0.008	. .	. .
2	−2	. .	+ 0.002	. .	. .
0	−3	. .	+ 0.024	− 0.001	. .
1	−3	− 0.324	+ 0.057	+ 0.001	. .
2	−3	+ 0.005	− 0.004	. .	. .
1	−4	. .	+ 0.004	. .	. .
4ω − 4ω					
1	−2	. .	+ 0.002	. .	. .
2	−2	− 0.033	+ 0.054	. .	. .
3	−2	− 0.018	+ 0.023	− 0.002	. .
0	−3	. .	+ 0.002	. .	. .
1	−3	+ 0.042	− 0.066	+ 0.035	. .
2	−3	− 0.350	+ 0.672	+ 0.269	− 0.002
3	−3	− 0.608	+ 0.917	+ 0.260	− 0.001
4	−3	− 0.236	+ 0.323	+ 0.085	. .
5	−3	− 0.023	+ 0.028	+ 0.010	. .
0	−4	− 0.026	+ 0.036	− 0.001	. .
1	−4	+ 0.886	+ 0.924	+ 0.013	− 0.010
2	−4	+ 30.040	− 47.590	+ 0.066	− 0.039
3	−4	+ 35.723	− 46.370	+ 0.077	− 0.040
4	−4	+ 10.683	− 12.120	+ 0.035	− 0.018
5	−4	+ 0.775	− 0.665	+ 0.006	− 0.003
6	−4	+ 0.048	− 0.033	. .	. .
7	−4	+ 0.003	− 0.002	. .	. .
0	−5	. .	+ 0.004	. .	. .
1	−5	+ 0.047	+ 0.079	− 0.020	− 0.001
2	−5	+ 2.666	− 4.311	− 0.246	− 0.004
3	−5	+ 4.140	− 5.541	− 0.259	− 0.007
4	−5	+ 1.508	− 1.834	− 0.094	− 0.004
5	−5	+ 0.118	− 0.112	− 0.013	. .
6	−5	+ 0.006	− 0.006	. .	. .
1	−6	. .	+ 0.004	− 0.002	. .
2	−6	[illegible]	− 0.254	− 0.026	. .
3	−6	+ 0.296	− 0.406	− 0.034	+ 0.001
4	−6	+ 0.125	− 0.158	− 0.013	. .
5	−6	+ 0.010	− 0.010	− 0.002	. .
2	−7	. .	− 0.012	− 0.001	. .
3	−7	+ 0.016	− 0.022	− 0.002	. .
4	−7	+ 0.008	− 0.009	. .	. .

TABLE I.—*Value of* $n\delta z$, *&c.*—Continued.

g	g'	$n\delta z$ sin.	$(n\delta z)^2$ cos.	$(n\delta z)^3$ sin.	$(n\delta z)^4$ cos.
$4\omega - 2\omega'$		″	″	″	″
2	−1	+ 0.002	− 0.005	− 0.008	. .
3	−1	+ 0.008	− 0.008	− 0.004	. .
4	−1	. .	− 0.002	. .	. .
1	−2	+ 0.015	− 0.146	− 0.004	. .
2	−2	− 1.092	+ 1.796	+ 0.024	. .
3	−2	− 0.602	+ 0.805	+ 0.026	. .
4	−2	+ 0.010	− 0.017	+ 0.008	. .
1	−3	+ 0.002	− 0.008	+ 0.001	. .
2	−3	− 0.044	+ 0.076	+ 0.009	. .
3	−3	− 0.041	+ 0.058	+ 0.008	. .
4	−3	+ 0.002	. .	+ 0.002	. .
2	−4	. .	+ 0.003	. .	. .
3	−4	. .	+ 0.003	. .	. .
$2\omega - 4\omega'$					
0	−3	. .	− 0.001	+ 0.001	. .
1	−3	− 0.023	+ 0.055	+ 0.009	. .
2	−3	− 0.012	+ 0.027	+ 0.004	. .
0	−4	+ 0.020	− 0.086	+ 0.001	. .
1	−4	+ 0.214	− 1.534	− 0.028	− 0.002
2	−4	+ 0.228	− 0.628	− 0.025	. .
3	−4	− 0.066	+ 0.090	− 0.006	. .
4	−4	− 0.005	+ 0.004	. .	. .
0	−5	. .	− 0.006	− 0.001	. .
1	−5	+ 0.021	− 0.162	− 0.011	. .
2	−5	+ 0.030	− 0.078	− 0.007	. .
3	−5	− 0.008	+ 0.010	− 0.002	. .
1	−6	. .	− 0.008	. .	. .
2	−6	. .	− 0.006	. .	. .
$5\omega - 5\omega'$					
3	−4	. .	− 0.009	− 0.006	. .
4	−4	. .	− 0.010	− 0.006	. .
5	−4	. .	− 0.002	− 0.001	. .
2	−5	. .	+ 0.026	+ 0.003	+ 0.001
3	−5	− 0.056	+ 0.090	+ 0.045	+ 0.001
4	−5	− 0.005	+ 0.042	+ 0.042	. .
5	−5	+ 0.007	− 0.001	+ 0.011	. .
2	−6	. .	+ 0.002	. .	. .
3	−6	− 0.006	+ 0.012	+ 0.004	. .
4	−6	− 0.001	+ 0.003	+ 0.005	. .
5	−6	+ 0.002	. .	+ 0.001	. .
$6\omega - 6\omega'$		″	″	″	″
2	−5	. .	− 0.002	− 0.001	. .
3	−5	− 0.004	+ 0.011	+ 0.011	− 0.003
4	−5	− 0.011	+ 0.028	+ 0.022	− 0.005
5	−5	− 0.008	+ 0.018	+ 0.017	− 0.003
6	−5	. .	+ 0.004	+ 0.002	. .
2	−6	+ 0.009	− 0.013	+ 0.015	. .
3	−6	+ 0.285	− 0.652	− 0.508	. .
4	−6	+ 0.538	− 1.082	− 0.749	. .
5	−6	+ 0.331	− 0.610	− 0.381	. .
6	−6	+ 0.084	− 0.138	− 0.070	. .
7	−6	+ 0.009	− 0.012	− 0.006	. .
2	−7	. .	− 0.001	+ 0.002	. .
3	−7	+ 0.037	− 0.088	− 0.070	+ 0.003
4	−7	+ 0.085	− 0.176	− 0.126	+ 0.005
5	−7	+ 0.061	− 0.116	− 0.076	+ 0.003
6	−7	+ 0.016	− 0.029	− 0.017	. .
7	−7	. .	− 0.002	− 0.001	. .
3	−8	. .	− 0.007	− 0.005	. .
4	−8	. .	− 0.016	− 0.012	. .
5	−8	. .	− 0.011	− 0.007	. .
6	−8	. .	− 0.002	− 0.001	. .
$6\omega - 4\omega'$					
2	−4	. .	− 0.001	− 0.002	. .
3	−4	− 0.011	+ 0.035	+ 0.028	. .
4	−4	− 0.010	+ 0.038	+ 0.027	. .
5	−4	− 0.005	+ 0.009	+ 0.006	. .
3	−5	. .	+ 0.003	− 0.003	. .
4	−5	. .	+ 0.005	+ 0.003	. .
$4\omega - 6\omega'$					
1	−6	. .	− 0.001	− 0.001	. .
2	−6	+ 0.003	− 0.017	− 0.030	. .
3	−6	+ 0.005	− 0.019	− 0.025	. .
4	−6	+ 0.001	− 0.004	− 0.004	. .
5	−6	. .	+ 0.002	+ 0.001	. .
2	−7	. .	− 0.002	− 0.004	. .
3	−7	. .	− 0.004	− 0.004	. .

TABLE II.—*Principal parts of* HANSEN'S *Ecliptic Longitude, with the Coefficients of the Concluded Longitude.*

g	g'	δS_0	$n\delta z \times$ $(e,g)_1 - 1$	$n\delta z \times$ $R_{1,1}$	$n\delta z \times$ δS_1	$(n\delta z)^2 \times$ $(e,g)_2$	$(n\delta z)^2 \times$ $R_{1,2}$	$(n\delta z)^3 \times$ $(e,g)_3$	Sum.	Terms in Ecliptic Longitude.	Principal Coefficient.
		″	″	″	″	″		″	″	″	
1	0	+ 1.103	− .253	− .169	+ .283	− 3.493	+ .003	. .	− 2.526	+ 22637.150	e
2	0	− 1.058	− .010	− .017	+ .044	− 1.766	. .	. .	− 2.807	+ 768.858	e^2
3	0	− .117	− .253	− .002	+ .005	− .252	. .	. .	− .619	+ 36.112	e^3
4	0	− .010	− .027	. .	. .	− .028	. .	. .	− .065	+ 1.931	e^4
5	0	. .	− .002	. .	. .	− .003	. .	. .	− .005	+ .113	.
6	0	. .	. .	. .	. .	. .	. .	. .	. .	+ 007	.
−5	−1	. .	+ .003	. .	. .	. .	. .	. .	+ .003	+ .003	.
−4	−1	. .	+ .038	. .	. .	+ .001	. .	. .	+ .039	+ .039	.
−3	−1	− .001	+ .515	. .	. .	+ .008	. .	. .	+ .522	+ .551	$e^3 e'$
−2	−1	− .012	+ 6.525	. .	− .003	+ .063	. .	− .004	+ 6.569	+ 7.666	$e e'$
−1	−1	− .022	+ 36.563	. .	− .012	+ .152	. .	− .007	+ 36.674	+ 109.908	$e e'$
0	−1	+ 2.187	+ 10.157	. .	+ .003	+ .045	. .	− .006	+ 12.386	+ 669.852	e'
1	−1	+ .034	+ 36.424	. .	+ .011	− .143	. .	− .007	+ 36.319	+ 148.000	$e e'$
2	−1	− .009	+ 8.625	. .	. .	− .107	. .	− .005	+ 8.504	+ 9.719	$e^2 e'$
3	−1	− .002	+ .665	. .	. .	− .018	. .	− .001	+ .644	+ 670	$e^3 e'$
4	−1	. .	+ .049	. .	. .	− .002	. .	. .	+ .047	+ .047	.
5	−1	. .	+ .003	. .	. .	. .	. .	. .	+ .003	+ .003	.
−3	−2	. .	+ .005	. .	. .	− .002	. .	. .	+ .003	+ .003	.
−2	−2	. .	+ .072	. .	. .	− .009	. .	. .	+ .063	+ .065	.
−1	−2	. .	+ .410	. .	. .	− .026	. .	. .	+ .384	+ 1.184	$e e'^2$
0	−2	+ .027	+ .163	. .	. .	− .002	. .	. .	+ .188	+ 7.507	e'^2
1	−2	. .	+ .407	. .	+ .001	+ .026	. .	. .	+ .434	+ 2.593	$e e'^2$
2	−2	. .	+ .146	. .	. .	+ .011	. .	. .	+ .157	+ .192	.
3	−2	. .	+ .012	. .	. .	+ .002	. .	. .	+ .014	+ .014	.
−2	−3	. .	+ .001	. .	. .	. .	. .	. .	+ .001	+ .001	.
−1	−3	. .	+ .004	. .	. .	. .	. .	. .	+ .004	+ .015	.
0	−3	. .	+ .003	. .	. .	. .	. .	. .	+ .003	+ .078	.
1	−3	. .	+ .004	. .	. .	. .	. .	. .	+ .004	+ .048	.
2	−3	. .	+ .003	. .	. .	. .	. .	. .	+ .003	+ .003	.
$2\omega - 2\omega'$											
−1	0	. .	− .015	. .	. .	. .	. .	. .	− .015	− .015	.
0	0	. .	− .139	. .	. .	+ .003	. .	. .	− .136	− .230	.
1	0	− .001	− .008	. .	. .	− .002	. .	. .	− .011	− 2.535	$e e'^2$
2	0	+ .006	− .139	. .	. .	− .003	. .	. .	− .136	− .188	.
3	0	. .	− .013	. .	. .	. .	. .	. .	− .013	− .013	.
−2	−1	. .	+ .004	. .	. .	+ .014	. .	. .	+ .018	+ .018	.
−1	−1	. .	+ .091	. .	. .	+ .126	. .	. .	+ .217	+ .177	.
0	−1	+ .004	− 1.605	. .	. .	+ .457	. .	. .	− 1.144	+ 2.521	$e^2 e'$
1	−1	− .046	− 1.067	. .	+ .002	+ .173	. .	− .001	− .939	− 28.559	$e e'$
2	−1	+ .494	− 1.576	+ .004	+ .037	− .405	. .	. .	− 1.446	− 24.452	e'
3	−1	+ .062	− 1.369	. .	+ .010	− .292	. .	. .	− 1.589	− 2.926	$e e'$
4	−1	+ .006	− .168	. .	+ .001	− .065	. .	. .	− .226	− .292	.
5	−1	. .	− .015	. .	. .	− .009	. .	. .	− .024	− .024	.
6	−1	. .	− .001	. .	. .	− .001	. .	. .	− .002	− .002	.
−1	−2	. .	+ .005	. .	. .	. .	. .	. .	+ .005	+ .005	.
−3	−2	. .	+ .071	. .	. .	. .	. .	. .	+ .071	+ .071	.
−2	−2	− .001	+ .080	. .	− .001	. .	. .	− .003	+ .075	+ .949	.

TABLE II.—*The Moon's Longitude*—Continued.

g	g'	$s\,S_0$	$n\delta z \times$ $(e,g)_1 - 1$	$n\delta z \times$ $R_{1,1}$	$n\delta z \times$ $s\,S_1$	$(n\delta z)^2 \times$ $(e,g)_2$	$(n\delta z)^2 \times$ $R_{1,2}$	$(n\delta z)^3 \times$ $(e,g)_3$	Sum.	Terms in Ecliptic Longitude.	Principal Coefficient.
$2\omega - 2\omega'$		″	″	″	″	″	″	″	″	″	
−1	−2	−.013	+15.102	. .	−.011	+.019	−.001	−.014	+15.082	+13.189	e^3
0	−2	+.137	+253.133	+.003	+.003	+.059	−.004	−.026	+253.305	+211.655	e^2
1	−2	+3.304	+115.652	. .	. .	+.032	−.002	−.024	+118.962	+4585.954	e
2	−2	−23.322	+248.293	−.166	+.016	−.041	−.004	−.028	+224.748	+2369.746	m^2
3	−2	−2.653	+134.633	−.006	−.003	−.046	−.003	−.023	+131.899	+191.921	e
4	−2	−.246	+12.569	+.001	−.006	−.015	−.001	−.008	+12.291	+14.374	e^2
5	−2	−.021	+1.003	. .	−.001	−.004	. .	−.001	+.976	+1.060	e^3
6	−2	−.002	+.077	. .	. .	. .	. .	. .	+.075	+.079	.
7	−2	. .	+.005	. .	. .	. .	. .	. .	+.005	+.005	.
−3	−3	. .	+.003	. .	. .	−.001	. .	. .	+.002	+.002	.
−2	−3	. .	+.043	. .	. .	−.012	. .	. .	+.031	+.031	.
−1	−3	−.001	+.658	. .	. .	−.099	. .	−.001	+.557	+.475	.
0	−3	+.004	+11.453	. .	. .	−.443	. .	−.002	+11.012	+8.660	e^2e'
1	−3	+.150	+8.399	. .	. .	−.217	. .	−.003	+8.329	+206.432	ee'
2	−3	−1.009	+11.147	−.007	−.035	+.377	−.001	−.002	+10.470	+165.517	e'
3	−3	−.126	+9.265	. .	−.011	+.306	. .	−.003	+9.431	+14.597	ee'
4	−3	−.012	+.921	. .	−.002	+.078	. .	−.001	+.954	+1.132	e^2e'
5	−3	. .	+.076	. .	. .	+.011	. .	. .	+.087	+.096	.
6	−3	. .	+.006	. .	. .	+.001	. .	. .	+.007	+.007	.
−2	−4	. .	+.002	. .	. .	. .	. .	. .	+.002	+.002	.
−1	−4	. .	+.023	. .	. .	−.005	. .	. .	+.018	+.018	.
0	−4	. .	+.415	. .	. .	−.026	. .	. .	+.389	+.280	.
1	−4	+.004	+.419	. .	. .	−.013	. .	. .	+.405	+7.440	ee'^2
2	−4	−.033	+.402	. .	−.002	+.020	. .	. .	+.387	+8.125	e'^2
3	−4	−.004	+.452	. .	. .	+.023	. .	. .	+.471	+.758	.
4	−4	. .	+.047	. .	. .	+.006	. .	. .	+.053	+.064	.
5	−4	. .	+.004	. .	. .	. .	. .	. .	+.004	+.004	.
0	−5	. .	+.013	. .	. .	−.001	. .	. .	+.012	+.012	.
1	−5	. .	+.018	. .	. .	−.001	. .	. .	+.017	+.257	.
2	−5	. .	+.014	. .	. .	+.001	. .	. .	+.015	+.344	.
3	−5	. .	+.019	. .	. .	+.001	. .	. .	+.020	+.032	.
4	−5	. .	+.002	. .	. .	. .	. .	. .	+.002	+.002	.
2ω											
1	2	+.002	. .	+.004	. .	. .	−.001	. .	+.005	+.005	.
2	2	−.011	. .	+.015	+.002	. .	−.002	. .	+.004	+.006	.
3	2	−.001	. .	+.004	. .	. .	−.001	. .	+.002	+.002	.
0	1	+.010	−.019	−.010	−.002	−.006	. .	. .	−.027	+.010	.
1	1	+.081	+.009	+.152	+.011	+.002	+.006	. .	+.264	−.087	.
2	1	−1.054	−.019	+1.341	+.004	+.006	+.012	. .	+.290	+.416	.
3	1	−.115	+.006	+.366	+.001	+.001	+.005	. .	+.264	+.265	.
4	1	−.010	. .	+.052	. .	. .	+.001	. .	+.043	+.043	.
5	1	. .	. .	+.006	. .	. .	. .	. .	+.006	+.006	.
−2	0	. .	+.003	. .	. .	−.001	. .	. .	+.002	+.002	.
−1	0	. .	+.001	. .	+.001	−.001	. .	. .	+.001	+.065	.
0	0	+1.180	−4.657	−.009	+.015	+.041	−.002	. .	−3.424	+1.083	I^2e^2
1	0	−.926	+.558	−.002	+.263	+.034	+.087	. .	+.014	−39.583	I^2e
2	0	−.053	−4.650	. .	+.130	−.042	+.264	. .	−4.351	−411.674	I^2
3	0	+.049	−.084	. .	+.005	−.032	+.158	. .	+.096	−45.091	I^2e
4	0	+.007	−.002	+.009	−.001	−.006	+.035	. .	+.042	−3.997	I^2e^2
5	0	. .	. .	+.002	. .	−.001	+.008	. .	+.009	−.329	.
6	0	. .	. .	. .	. .	. .	+.001	. .	+.001	−.026	.

TABLE II.—*The Moon's Longitude*—Continued.

g	g'	$s\,S_0$	$n\,\delta z \times$ $(e, g)_1 - 1$	$n\,\delta z \times$ $R_{1,1}$	$n\,\delta z \times$ $s\,S_1$	$(n\,\delta z)^2 \times$ $(e, g)_2$	$(n\,\delta z)^2 \times$ $R_{1,2}$	$(n\,\delta z)^3 \times$ $(e, g)_3$	Sum.	Terms in Ecliptic Longitude.	Principal Co-efficient.
2 ω		″	″	″	″	″	″	″	″	″	
−1	−1	. .	+ .004	. .	. .	+ .001	. .	. .	+ .005	+ .005	.
0	−1	− .007	+ .016	+ .006	+.002	+ .008	. .	. .	+ .025	+ .071	.
1	−1	− .144	+ .009	− .074	+.005	+ .002	+.003	. .	− .199	+ .080	.
2	−1	+ 1.104	+ .016	−1.324	+.005	− .009	+.010	. .	− .198	− .078	.
3	−1	+ .123	+ .008	− .444	−.001	− .002	+.009	. .	− .307	− .304	.
4	−1	+ .010	. .	− .065	. .	. .	+.002	. .	− .053	− .053	.
5	−1	. .	. .	− .007	. .	. .	. .	. .	− .007	− .007	.
1	−2	− .002	. .	− .001	. .	. .	. .	. .	− .003	− .003	.
2	−2	+ .016	. .	− .015	+.002	. .	−.002	. .	+ .001	+ .005	.
3	−2	+ .003	. .	− .007	. .	. .	−.001	. .	− .005	− .005	.
2 ω′											
−1	4	. .	− .007	. .	. .	. .	. .	. .	− .007	− .007	.
0	4	+ .034	. .	+ .015	−.001	. .	−.002	. .	+ .046	− .068	.
1	4	. .	− .007	+ .017	. .	. .	−.001	. .	+ .009	+ .009	.
2	4	. .	. .	+ .002	. .	. .	. .	. .	+ .002	+ .002	.
−3	3	. .	+ .002	. .	. .	. .	. .	. .	+ .002	+ .002	.
−2	3	. .	+ .020	. .	. .	− .004	. .	. .	+ .016	+ .028	.
−1	3	+ .014	− .190	− .006	−.008	− .010	−.002	. .	− .202	+ .402	.
0	3	+ 1.032	+ .025	+ .288	−.032	+ .001	−.018	. .	+ 1.296	− 2.153	e'
1	3	− .024	− .187	+ .446	+.006	+ .010	−.035	. .	+ .216	+ .064	.
2	3	− .008	− .022	+ .066	+.001	+ .003	−.011	. .	+ .029	+ .029	.
3	3	. .	− .002	+ .008	. .	. .	−.002	. .	+ .004	+ .004	.
−4	2	. .	+ .002	. .	. .	. .	. .	. .	+ .002	+ .002	.
−3	2	+ .001	+ .028	. .	. .	− .003	. .	. .	+ .026	+ .031	.
−2	2	+ .008	+ .291	− .003	−.005	− .028	. .	. .	+ .263	+ .425	.
−1	2	+ .094	− 4.504	− .117	−.001	− .049	. .	. .	− 4.577	+ 6.363	$I^2 e$
0	2	+23.660	+ .342	+3.794	+.016	+ .024	+.003	. .	+ 27.839	− 55.262	I^2
1	2	− .676	− 4.458	+9.628	. .	+ .052	+.005	. .	+ 4.551	− .175	.
2	2	− .275	− .564	+1.472	. .	+ .003	+.002	. .	+ .638	+ .561	.
3	2	− .028	− .044	+ .167	. .	. .	. .	. .	+ .095	+ .095	.
4	2	− .003	− .003	+ .018	. .	. .	. .	. .	+ .012	+ .012	.
5	2	. .	. .	+ .002	. .	. .	. .	. .	+ .002	+ .002	.
−2	1	. .	− .003	. .	. .	+ .001	. .	. .	− .002	− .008	.
−1	1	− .008	+ .113	. .	+.006	+ .007	+.002	. .	+ .120	− .075	.
0	1	− .505	− .013	− .044	+.034	+ .002	+.016	. .	− .510	+ 1.554	e'
1	1	− .020	+ .113	− .063	−.005	− .008	+.035	. .	+ .052	+ .009	.
2	1	− .003	+ .006	− .003	−.002	− .003	+.013	. .	+ .008	+ .008	.
3	1	. .	+ .001	. .	. .	. .	+.002	. .	+ .003	+ .003	.
0	0	− .006	. .	. .	. .	. .	. .	. .	− .006	+ .006	.
1	0	. .	. .	− .005	. .	. .	. .	. .	− .005	− .005	.
2 ω + 2 ω′											
2	3	− .003	. .	+ .007	. .	. .	. .	. .	+ .004	+ .004	.
3	3	. .	. .	+ .002	. .	. .	. .	. .	+ .002	+ .001	.
0	2	. .	+ .001	− .001	+.001	. .	. .	. .	+ .001	+ .001	.
1	2	+ .016	. .	− .031	−.004	. .	−.002	. .	− .021	− .034	.
2	2	− .081	− .001	+ .160	. .	. .	−.004	. .	+ .074	+ .075	.
3	2	− .008	. .	+ .036	. .	. .	−.001	. .	+ .027	+ .007	.
4	2	. .	. .	+ .005	. .	. .	. .	. .	+ .005	000	.
2	1	+ .0[illegible]	. .	− .004	. .	. .	. .	. .	− .003	− .003	.

TABLE II.—*The Moon's Longitude*—Continued.

g	g'	δS_0	$n\delta z \times$ $(e,g)_1 - 1$	$n\delta z \times$ R	$n\delta z \times$ δS	$(n\delta z)_2 \times$ $(e,g)_2$	$(n\delta z)_2 \times$ $(R_{1,2})$	$(n\delta z)^3 \times$ $(e,g)_3$	Sum.	Terms in Ecliptic Longitude.	Principal Coefficient.
$\omega-\omega'$		″	″				″	″	″	″	
0	1	. .	− .002	. .	. .	+ .002	. .	. .	.000	+ .007	.
1	1	. .	. .	. .	. .	. .	. .	. .	. .	− .031	.
2	1	. .	− .002	. .	. .	− .002	. .	. .	. .	− .004	.
−2	0	. .	+ .022	. .	. .	. .	. .	. .	− .004	− .004	.
−1	0	. .	+ .083	. .	. .	. .	. .	. .	+ .022	+ .022	.
0	0	+ .006	+ .981	. .	. .	− .004	. .	. .	+ .079	+ .369	−
1	0	. .	+ .033	. .	. .	− .010	. .	. .	+ .977	+ 1.293	π ee'
2	0	. .	+ .965	. .	. .	+ .002	. .	. .	+ .035	+ 17.601	π e'
3	0	. .	+ .080	. .	. .	− .011	. .	. .	+ .976	+ 1.235	π ee'
4	0	. .	+ .006	. .	. .	+ .003	. .	. .	+ .083	+ .083	.
−3	−1	. .	− .008	. .	. .	. .	. .	. .	+ .006	+ .006	.
−2	−1	. .	− .107	. .	. .	− .001	. .	. .	− .009	− .009	.
−1	−1	. .	− 1.083	. .	. .	− .011	. .	. .	− .118	− .118	.
0	−1	− .058	− 6.696	. .	. .	− .082	. .	. .	− 1.165	− 1.729	π e'
1	−1	− .047	− .717	+ .002	− .007	− .045	. .	+ .001	− 6.798	− 18.198	π e
2	−1	+ .020	− 6.704	. .	− .002	+ .071	. .	+ .001	− .697	− 122.032	π
3	−1	+ .002	− .549	. .	. .	+ .055	. .	+ .001	− 6.630	− 8.249	π e
4	−1	. .	− .041	. .	. .	+ .012	. .	. .	− .535	− .572	π e'
5	−1	. .	− .003	. .	. .	− .002	. .	. .	− .039	− .039	.
−2	−2	. .	− .001	. .	. .	. .	. .	. .	− .003	− .003	.
−1	−2	. .	− .010	. .	. .	− .002	. .	. .	+ .001	+ .001	.
0	−2	− .001	− .032	. .	. .	+ .007	. .	. .	− .003	− .012	.
1	−2	− .001	− .013	. .	. .	+ .013	. .	. .	− .020	− .167	.
2	−2	. .	− .032	. .	− .001	− .008	. .	. .	− .022	− .584	π e'
3	−2	. .	− .007	. .	. .	− .015	. .	. .	− .046	− .127	.
0	−3	. .	+ .002	. .	. .	− .004	. .	. .	− .011	− .017	.
1	−3	. .	. .	. .	. .	+ .001	. .	. .	+ .003	− .004	.
2	−3	. .	+ .002	. .	. .	− .001	. .	. .	− .001	+ .040	.
						− .001	. .	. .	+ .001	+ .001	.
$3\omega-3\omega'$											
0	−2	. .	− .001	. .	. .	− .002	. .	. .	− .003	− .003	.
1	−2	. .	+ .016	. .	. .	− .013	. .	. .	+ .003	− .035	.
2	−2	. .	+ .005	.	− .001	− .006	. .	. .	− .002	+ .270	.
3	−2	− .001	+ .016	. .	. .	+ .012	. .	. .	+ .027	+ .150	.
4	−2	. .	+ .008	. .	. .	+ .007	. .	. .	+ .015	+ .024	.
5	−2	. .	+ .001	. .	. .	+ .001	. .	. .	+ .002	+ .002	.
−1	−3	. .	− .005	. .	. .	+ .002	. .	. .	− .003	− .003	.
0	−3	. .	− .072	. .	. .	− .017	. .	. .	− .055	− .057	.
1	−3	− .001	− .171	. .	. .	− .080	. .	. .	− .092	− 1.184	π
2	−3	+ .009	− .026	.	− .001	+ .029	. .	. .	+ .011	− 3.143	π e
3	−3	+ .018	− .176	. .	+ .00[illegible]	− .074	. .	. .	− .226	+ .395	π
4	−3	+ .003	+ .022	. .	− .001	− .045	. .	. .	− .019	− .001	.
5	−3	. .	+ .003	. .	. .	− .008	. .	. .	− .005	− .004	.
0	−4	. .	− .005	. .	. .	− .001	. .	. .	− .004	− .004	.
1	−4	. .	− .013	. .	. .	− .005	. .	. .	− .008	− .074	.
2	−4	. .	+ .001	. .	. .	+ .002	. .	. .	+ .003	− .226	.
3	−4	. .	− .013	. .	. .	− .004	. .	. .	− .017	+ .061	.
4	−4	. .	+ .004	. .	. .	− .003	. .	. .	+ .001	+ .004	.
2	−5	. .	. .	. .	. .	. .	. .	. .	. .	− .012	.
3	−5	. .	− .001	. .	. .	. .	. .	. .	− .001	+ .006	.

Table 11.—*The Moon's Longitude*—Continued.

g		sS_0	$n\delta z\times$ $(e,g)_1-1$	$R_{1,1}$	sS_1	$(n\delta z)^2\times$ (e,g)	$R_{1,2}$	$(n\delta z)^3\times$ $(e,g)_3$	Sum.	Terms in Ecliptic Longitude.	Principal Coefficient.
$\omega+\omega'$		"	"	"	"	"				"	
2	2	+ .001	. .	. .	. .	. .	+ .001	. .	− .002	+ .002	.
−1	1	. .	+ .006	. .	. .	+ .001	. .	. .	− .007	+ .007	.
0	1	− .024	+ .042	+ .010	−.001	− .002	. .	. .	− .025	+ .075	.
1	1	+ .034	+ .003	− .243	−.006	− .001	−.003	. .	− .216	+ .541	.
2	1	+ .035	+ .042	− .063	. .	+ .002	−.006	. .	− .010	+ .010	.
3	1	+ .004	+ .003	− .010	. .	. .	−.002	. .	− .005	− .005	.
−1	0	. .	+ .003	. .	. .	. .	. .	. .	− .003	+ .003	.
0	0	. .	+ .002	− .002	. .	. .	. .	. .	.000	+ .049	.
1	0	. .	+ .003	+ .035	. .	. .	−.001	. .	− .037	+ .061	.
2	0	− .003	+ .002	+ .006	. .	. .	. .	. .	− .005	+ .005	.
$3\omega-\omega'$											
2	0	+ .003	. .	− .002	. .	. .	. .	. .	− .001	+ .001	.
3	0	. .	. .	− .035	. .	. .	−.001	. .	− .036	− .036	.
4	0	. .	. .	− .007	. .	. .	. .	. .	− .007	− .007	.
1	−1	− .003	+ .002	. .	. .	− .002	. .	. .	− .003	+ .003	.
2	−1	− .025	. .	+ .009	. .	. .	−.006	. .	− .022	+ .015	.
3	−1	− .010	+ .002	+ .246	. .	+ .002	−.005	. .	.235	+ .245	.
4	−1	− .001	+ .001	+ .043	. .	. .	−.001	. .	− .042	+ .042	.
5	−1	. .	. .	+ .005	. .	. .	. .	. .	− .005	+ .005	.
3	−2	− .001	. .	+ .001	. .	. .	+ .001	. .	− .001	+ .001	.
$\omega-3\omega'$											
−1	−3	+ .001	− .001	+ .003	. .	. .	+ .001	. .	− .004	+ .004	.
0	−3	+ .011	− .018	+ .007	. .	+ .002	−.005	. .	− .003	− .003	.
1	−3	+ .017	. .	− .002	−.006	. .	−.003	. .	− .006	− .318	.
2	−3	. .	− .018	+ .001	. .	− .002	. .	. .	− .019	− .014	.
3	−3	. .	− .001	. .	. .	. .	. .	. .	− .001	− .001	.
$4\omega-4\omega'$											
1	−2	. .	− .002	. .	. .	+ .002	. .	. .	.000	.000	.
2	−2	. .	− .001	. .	. .	+ .001	. .	. .	.000	− .033	.
3	−2	. .	− .002	. .	. .	− .002	. .	. .	− .004	− .022	.
4	−2	. .	− .001	. .	. .	− .001	. .	. .	− .002	− .002	.
0	−3	. .	+ .001	. .	. .	+ .001	. .	. .	− .002	+ .002	.
1	−3	. .	− .022	. .	. .	+ .022	. .	−[illegible]	− .003	+ .039	.
2	−3	. .	− .032	. .	. .	+ .028	. .	−[illegible]	− .006	− .356	.
3	−3	+ .007	− .032	. .	+.007	− .010	. .	−[illegible]	− .032	− .640	ee'
4	−3	+ .005	− .037	. .	+.005	− .027	.	−[illegible]	− .057	− .293	.
5	−3	+ .001	− .016	. .	. .	− .013	. .	−.001	− .029	− .052	.
6	−3	. .	− .003	. .	. .	− .002	. .	.	− .005	− .005	.
−2	−4	. .	+ .001	. .	. .	− .002	. .	. .	− .001	− .001	.
−1	−4	. .	+ .011	. .	. .	− .017	. .	. .	− .006	− .006	.
0	−4	. .	+ .172	. .	. .	− .174	. .	. .	− .002	− .028	.
1	−4	− .001	+ 1.785	. .	+.002	− 1.495	. .	. .	− .291	+ 1.177	e^3
2	−4	− .0[illegible]3	+ 2.050	. .	+.025	− 1.344	. .	. .	− .725	+ 30.768	e^2
3	−4	− .261	+ 2.242	. .	−.242	+ .968	−.004	. .	− 2.703	+ 38.426	e
4	−4	− .168	+ 2.117	. .	−.164	+ 1.434	−.002	. .	− 3.217	+ 13.900	m^4
5	−4	− .030	+ .732	. .	−.029	+ .533	. .	. .	− 1.206	+ 1.981	e
6	−4	− .003	+ .093	. .	−.003	+ .086	. .	. .	− .173	+ .221	.
7	−4	. .	+ .009	. .	. .	+ .011	. .	. .	− .020	+ .023	.

TABLE II.—*The Moon's Longitude*—Continued.

g	g'	$s\,S_0$	$n\delta z \times$ $(e, g)_1 - 1$	$R_{1,1}$	$s\,S_1$	$(n\delta z)^2 \times$ $(e, g)_2$	$R_{1,2}$	$(n\delta z)^3 \times$ $(e, g)_3$	Sum.	Terms in Ecliptic Longitude.	Principal Coefficient.
$4\omega - 4\omega'$		"	"	"	"	"	"	"	"	"	
−1	−5	. .	+ .001	. .	. .	− .002	. .	. .	− .001	− .001	.
0	−5	. .	+ .014	. .	. .	.017	. .	. .	− .003	− .003	.
1	−5	. .	+ .163	. .	. .	− .141	. .	+.003	+ .025	+ .072	.
2	−5	+ .001	+ .235	. .	+.002	− .161	. .	+.003	+ .080	+ 2.746	$e^2 e'$
3	−5	− .019	+ .230	. .	−.021	+ .068	. .	+.004	+ .262	+ 4.402	$e e'$
4	−5	− .018	+ .244	. .	−.017	+ .166	. .	+.003	+ .378	+ 1.886	e'
5	−5	− .003	+ .099	. .	−.003	+ .073	. .	+.002	+ .168	+ .286	.
6	−5	. .	+ .013	. .	. .	+ .012	. .	. .	+ .025	+ .031	.
7	−5	. .	+ .001	. .	. .	+ .001	. .	. .	+ .002	+ .002	.
1	−6	. .	+ .010	. .	. .	− .009	. .	. .	+ .001	+ .001	.
2	−6	. .	+ .017	. .	. .	− .012	. .	. .	+ .005	+ .156	.
3	−6	− .001	+ .016	. .	−.001	+ .003	. .	. .	+ .017	+ .313	.
4	−6	− 001	+ .017	. .	. .	+ .012	. .	. .	+ .028	+ .153	.
5	−6	. .	+ .008	. .	. .	+ .006	. .	. .	+ .014	+ .024	.
2	−7	. .	+ .001	. .	. .	− .001	. .	. .	.000	.000	.
3	−7	. .	+ .001	. .	. .	. .	. .	. .	+ .001	+ .017	.
4	−7	. .	+ .001	. .	. .	+ .001	. .	. .	+ .002	+ [illegible]	.
$4\omega - 2\omega'$											
3	0	. .	. .	+ .005	. .	. .	. .	. .	+ .005	+ .005	.
4	0	. .	. .	+ .001	. .	. .	. .	. .	+ .001	+ .001	.
2	−1	+ .002	+ .001	− .011	. .	. .	+.004	. .	− .004	− .002	.
3	−1	− .011	. .	+ .052	−.011	. .	+.032	. .	+ .062	+ .070	.
4	−1	− .009	. .	+ .055	−.007	. .	+.025	. .	+ .064	+ .064	.
5	−1	− .002	. .	+ .011	−.001	. .	+.007	. .	+ .015	+ .015	.
6	−1	. .	. .	+ .001	. .	. .	+.002	. .	+ .003	+ .003	.
−1	−2	. .	+ .001	. .	. .	. .	. .	. .	+ .001	+ .001	.
0	−2	− .001	− .003	. .	. .	+ .003	. .	. .	− .001	− .001	.
1	−2	− .012	− .063	. .	+.013	+ .053	. .	. .	− .009	+ .006	.
2	−2	− .012	− .032	+ .578	−.003	+ .026	+.001	. .	+ .558	− .534	.
3	−2	+ .001	− .060	−8.664	. .	− .050	+.004	. .	− 8.769	− 9.370	$I^2 e$
4	−2	+ .050	− .037	−5.744	. .	− .029	+.004	. .	− 5.756	− 5.743	I^2
5	−2	+ .011	− .002	−1.001	. .	− .003	+.001	. .	− .994	− .991	.
6	−2	+ .002	. .	− .126	. .	. .	. .	. .	− .124	− .124	.
7	−2	. .	. .	− .014	. .	. .	. .	. .	− .014	− .014	.
8	−2	. .	. .	− .001	. .	. .	. .	. .	− .001	− .001	.
1	−3	. .	− .003	. .	. .	+ .002	. .	. .	− .001	+ .001	.
2	−3	− .001	− .003	+ .027	−.001	+ .001	−.002	. .	+ .021	− .023	.
3	−3	+ .012	− .003	− .377	+.011	− .002	−.030	. .	− .389	− .430	.
4	−3	+ .011	− .003	− .374	+.007	− .002	−.025	. .	− .386	− .384	.
5	−3	+ .002	. .	− .070	+.001	. .	−.008	. .	− .075	− .075	.
6	−3	. .	. .	− .009	. .	. .	−.001	. .	− .010	− .010	.
7	−3	. .	. .	− .001	. .	. .	. .	. .	− .001	− .001	.
2	−4	. .	. .	+ .001	. .	. .	. .	. .	+ .001	+ .001	.
3	−4	. .	. .	− .013	. .	. .	−.002	. .	− .015	− .015	.
4	−4	+ .001	. .	− .016	. .	. .	−.002	. .	− .017	− .017	.
$2\omega - 4\omega'$											
0	−3	. .	− .002	+ .001	. .	+ .002	−.002	. .	− .001	− .001	.
1	−3	+ .006	. .	+ .001	−.008	+ .001	−.002	. .	− .002	− .025	.
2	−3	+ .004	− .001	. .	−.004	− .002	. .	. .	− .003	− .015	.

Table II.—*The Moon's Longitude*—Continued.

g	g'	εS_0	$n\delta z \times$ $(e,g)_1 - 1$	$n\delta z \times$ $R_{1,0}$	$n\delta z \times$ S_1	$(n\delta z)^2 \times$ $(e,g)_2$	$(n\delta z)^2 \times$ $R_{1,2}$	$(n\delta z)^3 \times$ $(e,g)_3$	Sum.	Terms in Ecliptic Longitude.	Principal Co-efficient.
$2\omega - 4\omega'$		″	″	″	″	″	″	″	″	″	
−2	−4	. .	. .	−.001	. .	. .	+.004	. .	+.003	+.003	.
−1	−4	−.004	. .	−.014	−.002	−.010	+.026	. .	−.004	−.004	.
0	−4	−.033	+.013	−.072	+.003	−.053	+.120	. .	−.022	−.002	.
1	−4	−.275	+.014	−.071	+.260	−.015	+.096	. .	+.009	+.223	.
2	−4	−.143	+.008	−.018	+.133	+.053	+.023	. .	+.056	−.001	.
3	−4	−.011	+.014	−.001	+.011	+.024	. .	. .	+.037	−.029	.
4	−4	−.001	−.003	−.002	. .	. .	. .	. .	−.006	−.011	.
5	−4	. .	. .	−.002	. .	. .	. .	. .	−.002	−.002	.
−1	−5	. .	. .	−.001	. .	. .	+.002	. .	+.001	+.001	.
0	−5	−.001	+.001	−.007	. .	−.005	+.011	. .	−.001	−.001	.
1	−5	−.021	+.002	−.008	+.023	−.002	+.012	. .	+.006	+.027	.
2	−5	−.016	+.001	−.003	+.015	+.005	+.003	. .	+.005	+.035	.
3	−5	−.001	+.002	. .	. .	+.003	. .	. .	+.004	−.004	.
1	−6	−.001	. .	. .	+.001	. .	. .	. .	.000	.000	.
$5\omega - 5\omega'$											
2	−5	. .	−.003	. .	. .	+.003	. .	. .	.000	.000	.
3	−5	. .	. .	. .	. .	+.001	. .	. .	+.004	−.055	.
4	−5	. .	−.003	. .	. .	−.003	. .	. .	−.006	−.011	.
5	−5	. .	. .	. .	. .	−.001	. .	. .	−.001	+.036	.
3	−6	. .	. .	. .	. .	. .	. .	. .	. .	−.006	.
4	−6	. .	. .	. .	. .	. .	. .	. .	. .	−.001	.
5	−6	. .	. .	. .	. .	. .	. .	. .	. .	+.002	.
$6\omega - 6\omega'$											
3	−5	. .	. .	. .	. .	+.001	. .	. .	+.001	−.003	.
4	−5	. .	. .	. .	. .	+.001	. .	. .	+.001	−.010	.
5	−5	. .	. .	. .	. .	−.001	. .	. .	−.001	−.009	.
1	−6	. .	+.002	. .	. .	−.004	. .	+.002	.000	.000	.
2	−6	. .	+.018	. .	. .	−.022	. .	+.006	+.002	+.011	.
3	−6	. .	+.031	. .	. .	−.032	. .	+.008	+.007	+.292	.
4	−6	−.003	+.034	. .	−.004	+.001	. .	+.008	+.036	+.572	.
5	−6	−.005	+.035	. .	−.004	+.028	. .	+.009	+.063	+.395	.
6	−6	−.002	+.021	. .	−.002	+.020	. .	+.005	+.042	+.126	.
7	−6	. .	+.006	. .	. .	+.007	. .	+.001	+.014	+.023	.
2	−7	. .	+.002	. .	. .	−.003	. .	+.001	.000	.000	.
3	−7	. .	+.005	. .	. .	−.006	. .	+.001	.000	+.037	.
4	−7	. .	+.006	. .	. .	−.003	. .	+.001	+.004	+.089	.
5	−7	. .	+.006	. .	. .	−.001	. .	+.001	+.006	+.067	.
6	−7	. .	+.004	. .	. .	. .	. .	. .	+.004	+.020	.
7	−7	. .	+.001	. .	. .	. .	. .	. .	+.001	+.001	.
$6\omega - 4\omega'$											
4	−3	. .	. .	+.001	. .	. .	+.002	. .	+.003	+.003	.
5	−3	. .	. .	+.001	. .	. .	+.002	. .	+.003	+.003	.
3	−4	. .	−.001	+.002	. .	+.001	+.004	. .	+.006	−.005	.
4	−4	. .	. .	−.056	. .	−.001	−.093	. .	−.150	−.166	.
5	−4	. .	−.001	−.080	. .	−.001	−.116	. .	−.198	−.203	.
6	−4	. .	. .	−.034	. .	. .	−.052	. .	−.086	−.086	.
7	−4	. .	. .	−.007	. .	. .	−.012	. .	−.019	−.019	.
8	−4	. .	. .	. .	. .	. .	−.002	. .	−.002	−.002	.

Table II.—*The Moon's Longitude*—Continued.

g	g'	δS_2	$n\,\delta z$			$(n\,\delta z)^2$		$(n\,\delta z)^3$	Sum.	Terms in Ecliptic Longitude.	Principal Coefficient.
			$(e,g)_1-1$	$R_{1,1}$	δS	$(e,g)_2$	$R_{1,2}$	$(e,g)_3$			
6 ω − 4 ω'			″	″	″	″	″				
4	−5	. .	. .	− .005	. .	. .	− .009	. .	− .014	− .014	.
5	−5	. .	. .	− .009	. .	. .	− .013	. .	− .022	− .022	.
6	−5	. .	. .	− .005	. .	. .	− .007	. .	− .012	− .012	.
7	−5	. .	. .	. .	. .	. .	− .002	. .	− .002	− .002	.
4 ω − 6 ω'											
1	−6	. .	. .	. .	. .	. .	+ .002	. .	+ .002	+ .001	.
2	−6	− .003	. .	− .001	+ .005	. .	+ .002	. .	+ .003	+ .003	.
3	−6	− .004	. .	− .001	+ .005	. .	+ .002	. .	+ .002	+ .005	.
4	−6	− .002	. .	. .	+ .002	. .	. .	. .	.000	+ .001	.
4 ω											
4	1	+ .002	. .	. .	. .	. .	. .	. .	+ .002	.000	.
2	0	+ .004	. .	− .021	. .	. .	. .	. .	− .017	− .010	.
3	0	+ .002	. .	+ .169	. .	. .	+ .003	. .	+ .174	+ .082	.
4	0	. .	. .	+ .020	. .	. .	+ .002	. .	+ .022	+ .422	.
5	0	. .	. .	+ .002	. .	. .	. .	. .	+ .002	+ .094	.
6	0	. .	. .	. .	. .	. .	. .	. .	. .	+ .013	.
4	1	− .002	. .	. .	. .	. .	. .	. .	− .002	.000	.
5	1	. .	. .	. .	. .	. .	. .	. .	. .	+ .001	.
4 ω'											
0	4	+ .002	. .	. .	− .004	. .	− .002	. .	− .004	− .004	.
2	4	. .	. .	. .	. .	. .	− .003	. .	− .003	− .003	.
6 ω − 2 ω'											
4	−2	. .	. .	+ .002	. .	. .	+ .003	. .	+ .005	+ .002	.
5	−2	. .	. .	+ .002	. .	. .	+ .002	. .	+ .004	+ .020	.
6	−2	. .	. .	. .	. .	. .	. .	. .	. .	+ .013	.
7	−2	. .	. .	. .	. .	. .	. .	. .	. .	+ .002	.
8 ω − 6 ω'											
5	−6	. .	. .	. .	. .	. .	− .002	. .	− .002	− .002	.
6	−6	. .	. .	− .001	. .	. .	− .002	. .	− .003	− .004	.
7	−6	. .	. .	. .	. .	. .	− .002	. .	− .002	− .003	.
5 ω − 3 ω'											
3	−3	. .	. .	+ .002	. .	. .	. .	. .	+ .002	+ .002	.
4	−3	. .	. .	+ .007	. .	. .	+ .006	. .	+ .013	+ .013	.
5	−3	. .	. .	. .	. .	. .	+ .004	. .	+ .004	+ .004	.
6	−3	. .	. .	. .	. .	. .	+ .001	. .	+ .001	+ .001	.

TABLE III.—*Reduced Coefficients of Longitude, according to* HANSEN *and* DELAUNAY

g	g'	*Hansen.*	*Delaunay* (1).	*Delaunay* (2).	$D_2 - D_1$	$H - D_2$
		"	"	"		
1	0	22640.15	22640.15	22640.15	.	.
2	0	+ 769.06	+ 769.12	+ 769.06	− 6	0
3	0	+ 36.13	+ 36.16	+ 36.12	− 4	+ 1
4	0	+ 1.94	+ 1.96	+ 1.94	− 2	0
5	0	+ 0.11	+ 0.12	+ 0.11	− 1	0
6	0	+ 0.01	+ 0.01	+ 0.01	0	0
−4	−1	+ 0.04	+ 0.04	. .	.	.
−3	−1	+ 0.55	+ 0.52	+ 0.56	+ 4	− 1
−2	−1	+ 7.67	+ 7.62	+ 7.69	+ 7	− 2
−1	−1	+ 109.92	+ 109.79	+ 109.85	+ 6	+ 7
0	−1	+ 669.85	+ 669.57	+ 669.76	+ 19	+ 9
1	−1	+ 148.02	+ 147.46	+ 148.43	+ 97	− 41
2	−1	+ 9.72	+ 9.59	+ 9.71	+ 12	+ 1
3	−1	+ 0.67	+ 0.63	+ 0.66	+ 3	+ 1
4	−1	+ 0.05	+ 0.04	. .	.	.
−2	−2	+ 0.06	+ 0.07	. .	.	.
−1	−2	+ 1.18	+ 1.16	+ 1.16	0	+ 2
0	−2	+ 7.51	+ 7.49	+ 7.46	− 3	+ 5
1	−2	+ 2.59	+ 2.49	+ 2.59	+ 10	0
2	−2	+ 0.19	+ 0.16	. .	.	.
3	−2	+ 0.01	+ 0.01	. .	.	.
−1	−3	+ 0.02	+ 0.02	. .	.	.
0	−3	+ 0.08	+ 0.14	. .	.	.
1	−3	+ 0.05	+ 0.03	. .	.	.
$2\omega - 2\omega'$						
−1	0	− 0.01	− 0.01	. .	.	.
0	0	− 0.23	− 0.16	. .	.	.
1	0	− 2.54	− 2.22	− 2.35	+ 13	+ 19
2	0	− 0.19	− 0.15	− 0.15	0	+ 4
3	0	− 0.01	− 0.01	. .	.	.
−2	−1	+ 0.02	. .	. .	.	.
−1	−1	+ 0.18	+ 0.07	. .	.	.
0	−1	+ 2.52	+ 1.87	+ 2.27	+ 40	+ 25
1	−1	− 28.56	− 29.50	− 28.32	−1.18	+ 24
2	−1	− 24.45	− 24.60	− 24.50	− 10	− 5
3	−1	− 2.93	− 2.96	− 2.96	0	− 3
4	−1	− 0.29	− 0.27	. .	.	.
5	−1	− 0.02	− 0.02	. .	.	.
−3	−2	+ 0.07	+ 0.06	. .	.	.
−2	−2	+ 0.95	+ 0.91	+ 1.00	+ 9	− 5
−1	−2	+ 13.19	+ 13.15	+ 13.32	+ 17	− 13
0	−2	+ 211.71	+ 211.46	+ 211.84	+ 38	− 13
1	−2	+ 4586.56	+ 4586.24	+ 4586.44	+ 20	+ 12
2	−2	+ 2369.75	+ 2369.74	+ 2369.74	0	+ 1
3	−2	+ 191.95	+ 192.00	+ 192.00	0	− 5
4	−2	+ 14.38	+ 14.40	+ 14.40	0	− 2
5	−2	+ 1.06	+ 1.06	+ 1.06	0	0
6	−2	+ 0.08	+ 0.08	. .	.	.

TABLE III.—*Reduced Coefficients of Longitude, &c.*—Continued.

g	*g′*	*Hansen.*	*Delaunay* (1).	*Delaunay* (2).	$D_2 - D_1$	$H - D_2$
2ω − 2ω′		″	″	″		
−2	−3	+ 0.03	+ 0.03	. .	.	.
−1	−3	+ 0.48	+ 0.49	+ 0.49	0	− 1
0	−3	+ 8.66	+ 8.66	+ 8.66	0	0
1	−3	+ 206.46	+ 206.54	+ 206.34	− 20	+ 12
2	−3	+ 165.52	+ 165.55	+ 165.55	0	− 3
3	−3	+ 14.60	+ 14.59	+ 14.66	+ 7	− 6
4	−3	+ 1.18	+ 1.11	+ 1.15	+ 4	+ 3
5	−3	+ 0.10	+ 0.08	. .	.	.
−1	−4	+ 0.02	+ 0.01	. .	.	.
0	−4	+ 0.28	+ 0.28	. .	.	.
1	−4	+ 7.44	+ 7.50	+ 7.50	0	− 6
2	−4	+ 8.13	+ 8.06	+ 8.06	0	+ 7
3	−4	+ 0.76	+ 0.68	+ 0.72	+ 4	+ 4
4	−4	+ 0.06	+ 0.05	. .	.	.
0	−5	+ 0.01	. .	. .	.	.
1	−5	+ 0.26	+ 0.19	. .	.	.
2	−5	+ 0.34	+ 0.25	. .	.	.
3	−5	+ 0.03	+ 0.01	. .	.	.
2ω						
0	1	+ 0.01	+ 0.02	. .	.	.
1	1	− 0.09	− 0.09	. .	.	.
2	1	+ 0.42	+ 0.42	+ 0.42	0	0
3	1	+ 0.27	+ 0.26	. .	.	.
4	1	+ 0.04	+ 0.04	. .	.	.
−1	0	+ 0.07	+ 0.05	+ 0.05	0	+ 2
0	0	+ 1.09	+ 1.39	+ 1.38	− 1	− 29
1	0	− 39.58	− 39.54	− 39.54	0	+ 4
2	0	− 411.60	− 411.63	− 411.63	0	− 3
3	0	− 45.09	− 45.12	− 45.12	0	− 3
4	0	− 4.00	− 4.01	− 4.01	0	− 1
5	0	− 0.33	− 0.33	− 0.33	0	0
6	0	− 0.03	− 0.03	. .	.	.
0	−1	+ 0.07	− 0.01	. .	.	.
1	−1	+ 0.08	+ 0.12	. .	.	.
2	−1	− 0.08	− 0.09	. .	.	.
3	−1	− 0.30	− 0.28	. .	.	.
4	−1	− 0.05	− 0.04	. .	.	.
5	−1	− 0.01	. .	. .	.	.
2ω′						
−1	4	− 0.01	+ 0.01	. .	.	.
0	4	− 0.07	− 0.07	. .	.	.
1	4	+ 0.01	. .	. .	.	.
−2	3	+ 0.03	+ 0.03	. .	.	.
−1	3	+ 0.40	+ 0.37	+ 0.37	0	+ 3
0	3	− 2.15	− 2.17	− 2.17	0	− 2
1	3	+ 0.06	+ 0.05	. .	.	.
2	3	+ 0.03	+ 0.02	. .	.	.

TABLE III.—*Reduced Coefficients of Longitude*, &c.—Continued.

g	g'	*Hansen.*	*Delaunay* (1).	*Delaunay* (2).	$D_2 - D_1$	$H - D_2$
$2\omega'$		″	″	″		
−3	2	+ 0.03	+ 0.03	. .	.	.
−2	2	+ 0.43	+ 0.45	+ 0.45	0	− 2
−1	2	+ 6.36	+ 6.37	+ 6.37	0	− 1
0	2	− 55.25	− 55.20	− 55.17	− 3	+ 8
1	2	− 0.18	− 0.18	− 0.14	− 4	+ 4
2	2	+ 0.56	+ 0.54	+ 0.54	0	+ 2
3	2	+ 0.10	+ 0.08	. .	.	.
4	2	+ 0.01	+ 0.01	. .	.	.
−2	1	− 0.01	− 0.01	. .	.	.
−1	1	− 0.08	− 0.10	. .	.	.
0	1	+ 1.55	+ 1.43	+ 1.43	0	+ 12
1	1	+ 0.01	+ 0.01	. .	.	.
2	1	+ 0.01	. .	. .	.	.
0	0	+ 0.01	+ 0.02	. .	.	.
$2\omega + 2\omega'$						
1	2	− 0.03	. .	. .	.	.
2	2	+ 0.08	+ 0.08	. .	.	.
3	2	+ 0.01	+ 0.02	. .	.	.
$\omega - \omega'$						
0	1	+ 0.01	. .	. .	.	.
1	1	− 0.03	− 0.04	. .	.	.
2	1	0.00	− 0.01	. .	.	.
−2	0	+ 0.02	+ 0.02	. .	.	.
−1	0	+ 0.38	+ 0.26	. .	.	.
0	0	+ 1.33	+ 0.87	+ 0.87	0	+ 46
1	0	+ 18.09	+ 18.08	+ 18.08	0	+ 1
2	0	+ 1.27	+ 1.22	+ 1.21	− 1	+ 6
3	0	+ 0.09	+ 0.09	. .	.	.
4	0	+ 0.01	+ 0.01	. .	.	.
−3	−1	− 0.01	. .	. .	.	.
−2	−1	− 0.12	− 0.09	. .	.	.
−1	−1	− 1.78	− 1.50	− 1.59	+ 9	+ 19
0	−1	− 18.70	− 18.35	− 18.76	+ 41	− 6
1	−1	− 125.43	− 125.49	− 125.98	+ 49	− 55
2	−1	− 8.48	− 8.45	− 8.54	+ 9	− 6
3	−1	− 0.59	− 0.57	− .60	+ 3	− 1
4	−1	− 0.04	− 0.04	. .	.	.
−1	−2	− 0.01	− 0.01	. .	.	.
0	−2	− 0.17	− 0.14	− 0.14	0	+ 3
1	−2	− 0.60	− 0.55	− 0.56	+ 1	+ 4
2	−2	− 0.13	− 0.08	. .	.	.
3	−2	− 0.02	− 0.01	. .	.	.
1	−3	+ 0.04	+ 0.05	. .	.	.
$3\omega - 3\omega'$						
1	−2	− 0.04	− 0.01	. .	.	.
2	−2	+ 0.28	+ 0.27	. .	.	.
3	−2	+ 0.15	+ 0.14	. .	.	.
4	−2	+ 0.02	+ 0.02	. .	.	.

TABLE III.—*Reduced Coefficients of Longitude, &c.*—Continued.

g	*g'*	*Hansen.*	*Delaunay* (1).	*Delaunay* (2).	$D_2 - D_1$	$H - D_2$
$3\omega - 3\omega'$		″	″	″		
0	−3	− 0.06	− 0.04	. .	.	.
1	−3	− 1.22	− 1.17	− 1.23	+ 6	− 1
2	−3	− 3.23	− 2.98	− 3.12	+ 14	+ 11
3	−3	+ 0.41	+ 0.57	+ 0.54	− 3	− 13
4	−3	. .	+ 0.04	+ 0.01	− 3	− 1
5	−3	. .	− 0.01	. .	.	.
1	−4	− 0.08	− 0.07	. .	.	.
2	−4	− 0.23	− 0.18	. .	.	.
3	−4	+ 0.06	+ 0.11	. .	.	.
4	−4	. .	+ 0.01	. .	.	.
2	−5	− 0.01	. .	. .	.	.
3	−5	+ 0.01	+ 0.01	. .	.	.
$\omega + \omega'$						
1	2	. .	+ 0.01	. .	.	.
−1	1	+ 0.01	. .	. .	.	.
0	1	+ 0.08	+ 0.04	. .	.	.
1	1	+ 0.55	+ 0.59	+ 0.59	0	− 4
2	1	+ 0.01	+ 0.03	. .	.	.
0	0	+ 0.05	. .	. .	.	.
1	0	+ 0.06	.	. .	.	.
$3\omega - \omega'$						
3	0	− 0.04	− 0.04	. .	.	.
4	0	− 0.01	− 0.01	. .	.	.
2	−1	+ 0.02	+ 0.02	. .	.	.
3	−1	+ 0.25	+ 0.24	.	.	.
4	−1	+ 0.04	+ 0.04	. .	.	.
$\omega - 3\omega'$						
1	−2	. .	+ 0.01	. .	.	.
0	−3	. .	− 0.03	. .	.	.
1	−3	− 0.32	− 0.26	− 0.25	− 1	+ 7
2	−3	− 0.01	− 0.01	. .	.	.
1	−4	. .	− 0.02	. .	.	.
$4\omega - 4\omega'$						
2	−2	− 0.03	− 0.01	. .	.	.
3	−2	− 0.02	− 0.01	. .	.	.
1	−3	+ 0.01	− 0.02	. .	.	.
2	−3	− 0.36	− 0.67	− 0.67	0	− 31
3	−3	− 0.64	− 0.83	− 0.83	0	− 19
4	−3	− 0.29	− 0.29	− 0.30	+ 1	− 1
5	−3	− 0.05	− 0.04	.	.	.
−1	−4	− 0.01	. .	. .	.	.
0	−4	− 0.03	. .	. .	.	.
1	−4	+ 1.18	+ 0.96	+ 1.08	+ 12	+ 10
2	−4	+ 30.78	+ 30.52	+ 30.72	+ 20	+ 6
3	−4	+ 38.43	+ 38.31	+ 38.48	+ 17	− 5
4	−4	+ 13.90	+ 13.89	+ 13.98	+ 9	− 8

TABLE III.—*Reduced Coefficients of Longitude, &c.*—Continued.

g	*g'*	*Hansen.*	*Delaunay* (1).	*Delaunay* (2)	$D_2 - D_1$	$H - D_2$
$4\omega - 4\omega'$		″	″	″		
5	−4	+ 1.98	+ 1.86	+ 1.88	+ 2	+ 10
6	−4	+ 0.22	+ 0.18	+ 0.20	+ 2	+ 2
7	−4	+ 0.02	+ 0.01	. .	.	.
1	−5	+ 0.07	+ 0.06	. .	.	.
2	−5	+ 2.75	+ 2.69	+ 2.75	+ 6	0
3	−5	+ 4.41	+ 4.28	+ 4.34	+ 6	+ 7
4	−5	+ 1.89	+ 1.67	+ 1.71	+ 4	+ 18
5	−5	+ 0.29	+ 0.20	. .	.	.
6	−5	+ 0.03	+ 0.01	. .	.	.
2	−6	+ 0.16	+ 0.11	. .	.	.
3	−6	+ 0.31	+ 0.22	. .	.	.
4	−6	+ 0.15	+ 0.10	. .	.	.
5	−6	+ 0.02	+ 0.01	. .	.	.
3	−7	+ 0.02	. .	. .	.	.
4	−7	+ 0.01	. .	. .	.	.
$4\omega - 2\omega'$						
2	−1	. .	+ 0.01	. .	.	.
3	−1	+ 0.07	+ 0.11	. .	.	.
4	−1	+ 0.06	+ 0.07	. .	.	.
5	−1	+ 0.02	+ 0.01	. .	.	.
1	−2	+ 0.01	. .	. .	.	.
2	−2	− 0.54	− 0.54	− 0.53	− 1	+ 1
3	−2	− 9.37	− 9.34	− 9.39	+ 5	− 2
4	−2	− 5.74	− 5.73	− 5.73	0	+ 1
5	−2	− 0.99	− 0.98	− 1.00	+ 2	− 1
6	−2	− 0.12	− 0.12	. .	.	
7	−2	− 0.01	− 0.01	. .	.	.
2	−3	− 0.02	− 0.02	. .	.	.
3	−3	− 0.43	− 0.43	− 0.43	0	0
4	−3	− 0.38	− 0.37	. .	.	.
5	−3	− 0.08	− 0.06	. .	.	.
6	−3	− 0.01	. .	. .	.	.
3	−4	− 0.02	− 0.01	. .	.	.
4	−4	− 0.02	− 0.01	. .	.	.
$2\omega - 1\omega'$						
1	−3	− 0.03	− 0.02	. .	.	.
2	−3	− 0.02	. .	. .	.	.
0	−4	. .	+ 0.01	. .	.	.
1	−4	+ 0.22	+ 0.34	+ 0.34	0	− 12
2	−4	. .	− 0.01	− 0.01	0	− 1
3	−4	− 0.03	− 0.06	. .	.	.
4	−4	− 0.01	− 0.01	. .	.	.
1	−5	+ 0.03	+ 0.03	. .	.	.
2	−5	+ 0.04	+ 0.01	+ 0.01	0	+ 3
$5\omega - 5\omega'$						
3	−5	− 0.00	− 0.02	. .	.	.
4	−5	− 0.01	+ 0.02	. .	.	.
5	−5	+ 0.01	+ 0.02	. .	.	.
3	−6	− 0.01	. .	. .	.	.

TABLE III.—*Reduced Coefficients of Longitude, &c.*—Continued.

g	*g'*		*Hansen.*		*Delaunay* (1).		*Delaunay* (2).	$D_2 - D_1$	$H - D_2$
$6\omega - 6\omega'$			"		"		"		
4	−5	−	0.01	−	0.01		. .	.	.
5	−5	−	0.01		. .		. .	.	.
2	−6	+	0.01		. .		. .	.	.
3	−6	+	0.29	+	0.20		. .	.	.
4	−6	+	0.57	+	0.40	+	0.51	+ 11	+ 6
5	−6	+	0.40	+	0.26		. .	.	.
6	−6	+	0.13	+	0.07		. .	.	.
7	−6	+	0.02	+	0.01		. .	.	.
3	−7	+	0.04	+	0.01		. .	.	.
4	−7	+	0.09	+	0.03		. .	.	.
5	−7	+	0.07	+	0.02		. .	.	.
6	−7	+	0.02		. .		. .	.	.
$6\omega - 4\omega'$									
3	−4	−	0.01	−	0.01		. .	.	.
4	−4	−	0.17	−	0.14		. .	.	.
5	−4	−	0.20	−	0.16		. .	.	.
6	−4	−	0.09	−	0.06		. .	.	.
7	−4	−	0.02	−	0.01		. .	.	.
4	−5	−	0.01	−	0.01		. .	.	.
5	−5	−	0.02	−	0.01		. .	.	.
6	−5	−	0.01		. .		. .	.	.
4ω									
2	0	−	0.01		. .		. .		
3	0	+	0.08	+	0.08		. .	.	.
4	0	+	0.42	+	0.42	+	0.42	0	0
5	0	+	0.09	+	0.09		. .	.	.
6	0	+	0.01	+	0.01		. .	.	.
$6\omega - 2\omega'$									
5	−2	+	0.02	+	0.02		. .	.	.
6	−2	+	0.01	+	0.01		. .	.	.
$5\omega - 3\omega'$									
4	−3	+	0.01	+	0.01		. .	.	.

TABLE IV.—*The Moon's Latitude.*

i	i'		sin I sin (f+ω)		s		sin β		β − sin β		β *Hansen.*		β *Delaunay* (1).		β *Delaunay* (2).
ω			″		″		″		″		″		″		″
0	3	−	.002		. .	−	.002		. .	−	.002		. .		. .
1	3	−	.003		. .	−	.003		. .	−	.003	−	.003		. .
2	3	−	.001		. .	−	.001		. .	−	.001		. .		. .
−1	2	−	.001		. .	−	.001		. .	−	.001	−	.004		. .
0	2	−	.092	−	.016	−	.108		. .	−	.108	−	.075	−	0.08
1	2	−	.317	+	.262	−	.055		. .	−	.055	−	.072	−	0.07
2	2	−	.064	+	.009	−	.055		. .	−	.055	−	.055	−	0.06
3	2	−	.006		. .	−	.006		. .	−	.006	−	.006	−	0.01
−2	1	−	.004	−	.020	−	.024		. .	−	.024	−	.024	−	0.02
−1	1	−	.071	−	.233	−	.304	−	.001	−	.305	−	.300	−	0.30
0	1	−	5.089	−	.573	−	5.662	−	.005	−	5.667	−	5.370	−	5.50
1	1	−	30.067	+	23.578	−	6.489	−	.008	−	6.497	−	6.471	−	6.33
2	1	−	6.610	+	1.279	−	5.331	−	.005	−	5.336	−	5.254	−	5.25
3	1	−	.720	+	.080	−	.640		. .	−	.640	−	.617	−	0.62
4	1	−	.068	+	.005	−	.063		. .	−	.063	−	.056	−	0.06
5	1	−	.004		. .	−	.004		. .	−	.004	−	.004		. .
−4	0		. .	−	.006	−	.006		. .	−	.006	−	.008	−	0.01
−3	0	−	.012	−	.080	−	.092		. .	−	.092	−	.095	−	0.09
−2	0	−	.254	−	1.328	−	1.582	−	.003	−	1.585	−	1.590	−	1.59
−1	0	−	6.933	−	24.787	−	31.720	−	.049	−	31.769	−	31.788	−	31.79
0	0	−	1020.614	+	21.919	−	998.695	−	.991	−	999.686	−	999.747	−	999.75
1	0	+	18444.607		0	+	18444.607	+	18.641	+	18463.248	+	18461.26	+	18461.26
2	0	+	1010.337	−	1.216	+	1009.121	+	1.052	+	1010.173	+	1010.233	+	1010.19
3	0	+	61.915	−	.055	+	61.860	+	.041	+	61.901	+	61.990	+	61.99
4	0	+	3.983	−	.003	+	3.980	−	.001	+	3.979	+	4.013	+	4.01
5	0	+	.263		. .	+	.263		. .	+	.263	+	.272	+	0.27
6	0	+	.019		. .	+	.019		. .	+	.019	+	.016	+	0.02
7	0	+	.001		. .	+	.001		. .	+	.001		. .		. .
−2	−1	+	.004	+	.021	+	.025		. .	+	.025	+	.024	+	0.02
−1	−1	+	.065	+	.246	+	.311	+	.001	+	.312	+	.316	+	0.32
0	−1	+	3.266	+	1.853	+	5.119	+	.006	+	5.125	+	5.014	+	5.07
1	−1	+	29.641	−	24.763	+	4.878	−	.002	+	4.880	+	4.955	+	4.80
2	−1	+	8.151	−	1.396	+	6.755	+	.008	+	6.763	+	6.519	+	6.62
3	−1	+	.780	−	.053	+	.727	+	.001	+	.728	+	.714	+	0.71
4	−1	+	.081	−	.005	+	.076		. .	+	.076	+	.064	+	0.06
5	−1	+	.005		. .	+	.005		. .	+	.005	+	.004		. .
0	−2	+	.040	+	.017	+	.057		. .	+	.057	+	.061	+	0.06
1	−2	+	.355	−	.336	+	.019		. .	+	.019	+	.037	+	0.04
2	−2	+	.142	−	.026	+	.116		. .	+	.116	+	.099	+	0.10
3	−2	+	.018		. .	+	.018		. .	+	.018	+	.011	+	0.01
4	−2	+	.001		. .	+	.001		. .	+	.001		. .		. .
1	−3	+	.004		. .	+	.004		. .	+	.004	+	.002		. .
2	−3	+	.003		. .	+	.003		. .	+	.003		. .		. .
… − 2ω															
−2	0	−	.002		. .	−	.002		. .	−	.002		. .		. .
−1	0	−	.015		. .	−	.015		. .	−	.015	−	.007	−	0.01
0	0	−	.111	+	.007	−	.104		. .	−	.104*	−	.083	−	0.08
1	0	−	.003	−	.128	−	.131		. .	−	.131	−	.122	−	0.12
2	0		. .	−	.005	−	.005		. .	−	.005	−	.006	−	0.01

Table IV.—*The Moon's Latitude*—Continued.

g	g'	sin I sin ($f + \omega$)		sin β	β − sin β	β *Hansen.*	β *Delaunay* (1).	β *Delaunay* (2).
ω − 2ω'		″	″	″	″	″	″	″
−3	−1	+ .004	. .	+ .004	. .	+ .004	− .001	. .
−2	−1	+ .026	. .	+ .026	. .	+ .026	− .011	− 0.01
−1	−1	+ .151	− .062	+ .089	. .	+ .089	− .073	− 0.07
0	−1	− .961	+ .164	− .797	. .	− .797	− 1.105	− 1.00
1	−1	− .940	− 11.183	− 12.123	− .017	− 12.140	− 12.179	− 12.18
2	−1	− .041	− .789	− .830	− .002	− .832	− .820	− 0.82
3	−1	− .003	− .058	− .061	. .	− .061	− .060	− 0.06
−5	−2	+ .001	. .	+ .001	. .	+ .001	. .	. .
−4	−2	+ .013	. .	+ .013	. .	+ .013	+ .007	+ 0.01
−3	−2	+ .153	− .019	+ .134	. .	+ .134	+ .116	+ 0.12
−2	−2	+ 1.822	− .303	+ 1.519	. .	+ 1.519	+ 1.459	+ 1.49
−1	−2	+ 21.041	− 5.490	+ 15.551	+ .009	+ 15.560	+ 15.362	+ 15.51
0	−2	+ 210.540	− 44.062	+ 166.478	+ .125	+ 166.603	+ 166.305	+ 166.57
1	−2	+ 99.960	+ 522.584	+ 622.544	+ 1.158	+ 623.702	+ 623.622	+ 623.62
2	−2	+ 2.716	+ 30.588	+ 33.304	+ .065	+ 33.369	+ 33.382	+ 33.38
3	−2	+ .116	+ 2.027	+ 2.143	+ .003	+ 2.146	+ 2.160	+ 2.16
4	−2	+ .005	+ .140	+ .145	. .	+ .145	+ .149	+ 0.15
5	−2	. .	+ .009	+ .009	. .	+ .009	+ .011	+ 0.01
−3	−3	+ .005	. .	+ .005	. .	+ .005	+ .003	. .
−2	−3	+ .063	− .007	+ .056	. .	+ .056	+ .052	+ 0.05
−1	−3	+ .826	− .171	+ .655	. .	+ .655	+ .719	+ 0.79
0	−3	+ 9.249	− 1.779	+ 7.470	+ .005	+ 7.475	+ 7.462	+ 7.50
1	−3	+ 6.910	+ 22.773	+ 29.683	+ .053	+ 29.736	+ 29.683	+ 29.68
2	−3	+ .198	+ 1.574	+ 1.772	+ .004	+ 1.776	+ 1.754	+ 1.75
3	−3	+ .009	+ .112	+ .121	. .	+ .121	+ .124	+ 0.12
−2	−4	+ .002	. .	+ .002	. .	+ .002	+ .001	. .
−1	−4	+ .026	. .	+ .026	. .	+ .026	+ .019	+ 0.02
0	−4	+ .330	− .054	+ .276	. .	+ .276	+ .256	+ 0.26
1	−4	+ .337	+ .756	+ 1.093	+ .002	+ 1.095	+ 1.083	+ 1.08
2	−4	+ .011	+ .052	+ .063	. .	+ .063	+ .065	+ 0.06
−1	−5	+ .001	. .	+ .001	. .	+ .001	. .	. .
0	−5	+ .011	. .	+ .011	. .	+ .011	+ .004	. .
1	−5	+ .014	. .	+ .014	. .	+ .014	+ .028	+ 0.03
ω + 2ω'								
1	4	− .005	. .	− .005	− .001	− .004	− .003	. .
0	3	+ .029	− .002	+ .027	− .002	+ .025	+ .016	+ 0.02
1	3	− .142	+ .026	− .116	+ .028	− .088	− .091	− 0.09
2	3	− .021	− .001	− .022	+ .010	− .012	− .012	− 0.01
3	3	− .002	. .	− .002	+ .002	0	− .001	. .
−1	2	+ .005	+ .003	+ .008	− .002	+ .006	+ .011	+ 0.01
0	2	+ .506	− .186	+ .320	− .034	+ .286	+ .291	+ 0.29
1	2	− 3.565	+ .768	− 2.797	+ .603	− 2.194	− 2.190	− 2.19
2	2	− .801	+ .243	− .558	+ .232	− .320	− .313	− 0.31
3	2	− .063	. .	− .063	+ .040	− .023	− .023	− 0.02
4	2	− .006	. .	− .006	+ .005	− .001	− .002	. .
0	1	− .009	. .	− .009	+ .001	− .008	− .005	− 0.01
1	1	+ .085	− .008	+ .077	− .012	+ .065	+ .054	+ 0.05
2	1	+ .006	. .	+ .006	− .002	+ .004	+ .005	. .
3	1	+ .001	. .	+ .001	. .	+ .001	. .	. .

Table IV.—*The Moon's Latitude*—Continued.

g	g'	sin I sin $(f+\omega)$	s	sin β	β − sin β	β *Hansen.*	β *Delaunay* (1).	β *Delaunay* (2).
$\omega - 4\omega$		"	"	"	"	"	"	"
−2	−4	− .002	. .	− .002	. .	− .002	. .	. .
−1	−4	− .010	. .	− .010	+ .002	− .008	− .002	. .
0	−4	− .033	+ .004	− .029	+ .005	− .024	− .002	. .
1	−4	− .004	+ .054	+ .050	+ .018	+ .068	+ .064	+ 0.06
2	−4	− .001	. .	− .001	+ .001	0	− [illegible]	. .
−1	−5	− .001	. .	− .001	. .	− .001	. .	. .
0	−5	− .003	. .	− .003	+ .001	− .002	. .	. .
1	−5	. .	− .003	− .003	+ .002	− .001	+ .006	+ 0.01
3ω								
1	1	+ .001	. .	+ .001	. .	+ .001	. .	. .
2	1	− .011	. .	− .011	+ .004	− .007	− .006	− 0.01
3	1	+ .005	. .	+ .005	+ .008	+ .013	+ .013	+ 0.01
4	1	. .	. .	. .	+ .006	+ .006	+ .007	+ 0.01
5	1	. .	. .	. .	+ .001	+ .001	+ .001	. .
0	0	+ .005	+ .003	+ .008	− .001	+ .007	+ .001	. .
1	0	+ .268	− .137	+ .131	− .023	+ .108	+ .133	+ 0.13
2	0	− 3.818	+ .002	− 3.816	+ 1.010	− 2.806	− 2.697	− 2.70
3	0	− .252	. .	− .252	− 6.051	− 6.303	− 6.297	− 6.30
4	0	− .021	. .	− .021	− 1.000	− 1.021	− 1.018	− 1.02
5	0	− .002	. .	− .002	− .117	− .11[illegible]	− .119	− 0.12
6	0	. .	. .	. .	− .012	− .012	− .012	− 0.01
7	0	. .	. .	. .	− .001	− .001	. .	. .
1	−1	+ .002	. .	+ .002	. .	+ .002	. .	. .
2	−1	+ .006	. .	+ .006	− .004	+ .002	+ .008	+ 0.01
3	−1	+ .003	. .	+ .003	− .003	0	. .	. .
4	−1	. .	. .	. .	− .007	− .007	− .006	− 0.01
5	−1	. .	. .	. .	− .001	− .001	− .001	. .
$3\omega - 2\omega'$								
1	0	− .005	. .	− .005	. .	− .005	− .001	. .
2	0	− .116	. .	− .116	. .	− .116	− .090	− 0.09
3	0	− .015	. .	− .015	. .	− .015	− .009	− 0.01
4	0	− .001	. .	− .001	. .	− .001	. .	. .
0	−1	− .002	+ .003	+ .001	. .	+ .001	− .001	. .
1	−1	+ .112	− .052	+ .060	. .	+ .060	+ .021	+ 0.02
2	−1	− [illegible]	+ .256	− 1.318	− .003	− 1.321	− 1.802	− 1.50
3	−1	− 1.430	+ .150	− 1.[illegible]	+ .003	− 1.277	− 1.382	− 1.38
4	−1	− .259	+ .017	− .242	+ .002	− .240	− .239	− 0.24
5	−1	− .034	. .	− .034	. .	− .034	− .023	− 0.02
6	−1	− .003	. .	− .003	. .	− .003	. .	. .
−1	−2	+ .005	+ .029	+ .034	. .	+ .034	+ .025	+ 0.03
0	−2	− .004	+ .273	+ .269	. .	+ .269	+ .246	+ 0.25
1	−2	− 1.85[illegible]	+ .236	− 1.622	− .001	− 1.623	− 1.739	− 1.68
2	−2	+ 199.476	− .296	+ 199.180	+ .303	+ 199.483	+ 199.277	+ 199.42
3	−2	+ 117.753	− .096	+ 117.657	− .399	+ 117.258	+ 117.188	+ 117.19
4	−2	+ 15.207	− .015	+ 15.192	− .077	+ 15.115	+ 15.105	+ 15.11
5	−2	+ 1.531	− .002	+ 1.529	− .010	+ 1.519	+ 1.502	+ 1.50
6	−2	+ .141	. .	+ .141	− .001	+ .140	+ .132	+ 0.13
7	−2	+ .012	. .	+ .012	. .	+ .012	+ .008	+ 0.01

TABLE IV.—*The Moon's Latitude*—Continued.

g	g'	$\sin I \sin (f+\omega)$	s	$\sin \beta$	$\beta - \sin \beta$	β *Hansen.*	β *Delaunay* (1).	β *Delaunay* (2).
$3\omega - 2\omega'$		″		″	″	″	″	″
0	−3	. .	+ .010	+ .010	. .	+ .010	+ .010	+ 0.01
1	−3	− .071	+ .010	− .061	. .	− .061	− .078	− 0.08
2	−3	+ 9.184	− .286	+ 8.898	+ .014	+ 8.912	+ 8.968	+ 9.00
3	−3	+ 8.180	− .168	+ 8.012	− .015	+ 7.997	+ 7.946	+ 7.95
4	−3	+ 1.166	− .022	+ 1.144	− .004	+ 1.140	+ 1.082	+ 1.08
5	−3	+ .128	. .	+ .128	− .001	+ .127	+ .100	+ 0.10
6	−3	+ .013	. .	+ .013	. .	+ .013	+ .006	+ 0.01
1	−4	− .003	.	− .003	. .	− .003	− .003	. .
2	−4	+ .334	− .014	+ .320	. .	+ .320	+ .311	+ 0.31
3	−4	+ .401	− .011	+ .390	. .	+ .390	+ .362	+ 0.36
4	−4	+ .061	. .	+ .061	. .	+ .061	+ .043	+ 0.04
5	−4	+ .007	. .	+ .007	. .	+ .007	+ .002	. .
2	−5	+ .012	. .	+ .012	. .	+ .012	+ .005	+ 0.01
3	−5	+ .017	. .	+ .017	. .	+ .017	+ .006	+ 0.01
4	−5	+ .002	. .	+ .002	. .	+ .002	. .	. .
$3\omega - 4\omega'$								
2	−2	− .001	. .	− .001	. .	− .001	− .002	. .
0	−3	+ .001	. .	+ .001	. .	+ .001	. .	. .
1	−3	− .003	+ .002	− .001	. .	− .001	− .014	− 0.01
2	−3	− .010	− .143	− .153	. .	− .153	− .199	− 0.20
3	−3	− .005	− .098	− .103	. .	− .103	− .114	− 0.11
4	−3	− .001	− .012	− .013	. .	− .013	− .014	− 0.01
−2	−4	− .003	. .	− .003	. .	− .003	. .	. .
−1	−4	− .012	. .	− .012	. .	− .012	− .002	. .
0	−4	− .043	+ .045	+ .002	. .	+ .002	− .005	. .
1	−4	+ .220	+ .404	+ .624	+ .001	+ .625	+ .582	+ 0.61
2	−4	+ .603	+ 5.957	+ 6.560	+ .016	+ 6.576	+ 6.532	+ 6.56
3	−4	+ .236	+ 3.451	+ 3.687	− .008	+ 3.679	+ 3.651	+ 3.65
4	−4	+ .020	+ .448	+ .468	− .002	+ .466	+ .464	+ 0.46
5	−4	+ .001	+ .047	+ .048	. .	+ .048	+ .044	+ 0.04
−1	−5	− .002	. .	− .002	. .	− .002	. .	. .
0	−5	− .005	. .	− .005	. .	− .005	. .	. .
1	−5	+ .018	+ .011	+ .029	. .	+ .029	+ .042	+ 0.04
2	−5	+ .066	+ .450	+ .516	+ .001	+ .517	+ .576	+ 0.60
3	−5	+ .031	+ .379	+ .410	. .	+ .410	+ .395	+ 0.40
4	−5	+ .003	+ .050	+ .053	. .	+ .053	+ .046	+ 0.05
2	−6	+ .004	+ .021	+ .025	. .	+ .025	+ .028	+ 0.03
3	−6	+ .002	+ .019	+ .021	. .	+ .021	+ .022	+ 0.02
$5\omega - 6\omega'$								
1	−6	− .001	. .	− .001	. .	− .001	. .	. .
2	−6	− .002	+ .005	+ .003	. .	+ .003	+ .003	. .
3	−6	+ .005	+ .068	+ .073	. .	+ .073	+ .061	+ 0.06
4	−6	+ .005	+ .086	+ .091	. .	+ .091	+ .071	+ 0.07
5	−6	+ .001	+ .035	+ .036	. .	+ .036	+ .024	+ 0.02
6	−6	. .	+ .002	+ .002	. .	+ .002	+ .002	. .
4	−7	+ .001	. .	+ .001	. .	+ .001	+ .005	. .

Table IV.—*The Moon's Latitude*—Continued.

i	i′		sin I sin(f+ω)		s		sin β		β − sin β		β Hansen		β Delaunay (1)		β Delaunay (2).
— ω			″		″		″		″				″		″
−3	0	+	.002		. .	+	.002		. .	+	.002	−	.001		. .
−2	0	+	.022		. .	+	.022		. .	+	.022	−	.010	+	0.01
−1	0	+	.096	−	.080	+	.016		. .	+	.016	−	.024	+	0.02
0	0	+	.777	+	.015	+	.792	+	.001	+	.793	−	.794	+	0.79
1	0	+	.009	+	.004	+	.013		. .	+	.013	−	.035	+	0.03
−4	−1	−	.001		. .	−	.001		. .	−	.001		. .		. .
−3	−1	−	.015		. .	−	.015		. .	−	.015	−	.005	−	0.01
−2	−1	−	.151	+	.041	−	.110		. .	−	.110	−	.080	−	0.08
−1	−1	−	1.174	+	.751	−	.423		. .	−	.423	−	.381	−	0.38
0	−1	−	5.504	+	.821	−	4.683	−	.005	−	4.688	−	4.756	−	4.83
1	−1	−	.083	−	.500	−	.583	−	.001	−	.584	−	.603	−	0.60
2	−1	−	.004	−	.030	−	.034		. .	−	.034	−	.039	−	0.04
−2	−2	+	.001		. .	+	.001		. .	+	.001	−	.002		. .
1	−2		. .	+	.019	+	.019		. .	+	.019	−	.010	+	0.01
[illegible]	−2	−	.015	−	.005	−	.020		. .	−	.020	−	.005	−	0.01
1	−2	−	.002	−	.008	−	.010		. .	−	.010	−	.018	−	0.02
0	−3	+	.002		. .	+	.002		. .	+	.002	−	.002		. .
2 .. — l′															
2	1	−	.002		.	−	.002		. .	−	.002	−	.002		. .
0	0	+	.013	+	.004	+	.017		. .	+	.017	−	.014	+	0.01
1	0	+	.018	−	.069	−	.051		. .	−	.051	−	.043	−	0.04
2	0	+	.796	−	.009	+	.787	+	.001	+	.788	−	.791	+	0.79
3	0	+	.101		. .	+	.101		. .	+	.101	−	.101	+	0.10
4	0	+	.010		. .	+	.010		. .	+	.010	−	.008	+	0.01
0	−1	−	.023	−	.043	−	.066		. .	−	.066	−	.062	−	0.06
1	−1	−	.452	+	.577	+	.125		. .	+	.125	−	.141	+	0.14
2	−1	−	5.431	+	.186	−	5.245	−	.006	−	5.251	−	5.325	−	5.40
3	−1	−	.659	+	.009	−	.650		. .	−	.650	−	.647	−	0.67
4	−1	−	.063		. .	−	.063		. .	−	.063	−	.055	−	0.06
5	−1	−	.006		. .	−	.006		. .	−	.006	−	.004		. .
1	−2	−	.014	+	.009	−	.005		. .	−	.005		. .		. .
2	−2	−	.038	+	.026	−	.012		. .	−	.012	−	.013	−	0.01
3	−2	−	.011		. .	−	.011		. .	−	.011	−	.004		. .
4	−2	−	.002		. .	−	.002		. .	−	.002		. .		. .
2	−3	+	.002		. .	+	.002		. .	+	.002	−	.002		. .
2 . — 3 .															
0	−2	−	.002		. .	−	.002		. .	−	.002		. .		. .
1	−2	+	.003		. .	+	.003		. .	+	.003	−	.012	+	0.01
2	−2	+	.001	+	.020	+	.021		. .	+	.021	−	.033	+	0:03
−1	−3	−	.004		. .	−	.004		. .	−	.004	−	.002		. .
0	−3	−	.046	−	.002	−	.048		. .	−	.048	−	.034	−	0.03
1	−3	−	.071	−	.224	−	.295		. .	−	.295	−	.290	−	0.29
2	−3	+	.041	−	.391	−	.350	−	.001	−	.351	−	.339	−	0.34
3	−3	+	.001	−	.045	−	.044		. .	−	.044	−	.031	−	0.03
0	−4	−	.003		. .	−	.003		. .	−	.003	−	.001		. .
1	−4	−	.006		. .	−	.006		. .	−	.006	−	.017	−	0.02
2	−4	−	.006		. .	+	.006		. .	+	.006	−	.021	−	0.02

Table IV.—*The Moon's Latitude*—Continued.

g	g'	sin I sin $(l+\omega)$	s	sin β	β − sin β	β *Hansen.*	β *Delaunay* (1).	β *Delaunay* (2).
$5\omega - 4\omega'$		"	"	"	"	"	"	"
3	−2	− .002	. . .	− .002	. .	− .002	. .	. .
4	−2	− .001	. . .	− .001	. .	− .001	. .	. .
2	−3	+ .005	. . .	+ .005	. .	+ .005	. .	. .
3	−3	− .030	. . .	− .030	. .	− .030	− .058	− 0.06
4	−3	− .055	. . .	− .055	. .	− .055	− .058	− 0.06
5	−3	− .027	. . .	− .027	. .	− .027	− .020	− 0.02
6	−3	− .005	. . .	− .005	. .	− .005	− .001	. .
1	−4	− .003	. . .	− .003	. .	− .003	. .	. .
2	−4	+ .019	. . .	+ .019	. .	+ .019	+ .006	+ 0.01
3	−4	+ 2.415	. . .	+ 2.415	+ .004	+ 2.419	+ 2.257	+ 2.32
4	−4	+ 3.017	. . .	+ 3.017	− .013	+ 3.004	+ 2.816	+ 2.89
5	−4	+ 1.204	. . .	+ 1.204	− .011	+ 1.193	+ 1.079	+ 1.09
6	−4	+ .216	. . .	+ .216	− .002	+ .214	+ .162	+ 0.16
7	−4	+ .029	. . .	+ .029	. .	+ .029	+ .012	+ 0.01
8	−4	+ .003	. . .	+ .003	. .	+ .003	. .	. .
3	−5	+ .218	. . .	+ .218	. .	+ .218	+ .168	+ 0.17
4	−5	+ .347	. . .	+ .347	− .001	+ .346	+ .256	+ 0.26
5	−5	+ .162	. . .	+ .162	− .001	+ .161	+ .102	+ 0.10
6	−5	+ .031	. . .	+ .031	. .	+ .031	+ .010	+ 0.01
7	−5	+ .003	. . .	+ .003	. .	+ .003	. .	. .
3	−6	+ .012	. . .	+ .012	. .	+ .012	+ .005	+ 0.01
4	−6	+ .024	. . .	+ .024	. .	+ .024	+ .008	+ 0.01
5	−6	+ .013	. . .	+ .013	. .	+ .013	+ .003	. .
6	−6	+ .002	. . .	+ .002	. .	+ .002	. .	. .
4	−7	+ .001	. . .	+ .001	. .	+ .001	. .	. .
$4\omega - 3\omega'$								
2	−2	− .002	. . .	− .002	. .	− .002	. .	. .
3	−2	+ .021	. . .	+ .021	. .	+ .021	+ .018	+ 0.02
4	−2	+ .014	. . .	+ .014	. .	+ .014	+ .011	+ 0.01
5	−2	+ .002	. . .	+ .002	. .	+ .002	+ .002	. .
2	−3	− .054	. . .	− .054	. .	− .054	− .043	− 0.04
3	−3	− .205	. . .	− .205	. .	− .205	− .165	− 0.17
4	−3	− .030	. . .	− .030	+ .001	− .029	+ .005	+ 0.01
5	−3	− .007	. . .	− .007	. .	− .007	+ .005	+ 0.01
6	−3	− .001	. . .	− .001	. .	− .001	+ .001	. .
2	−4	− .003	. . .	− .003	. .	− .003	− .002	. .
3	−4	− .014	. . .	− .014	. .	− .014	− .005	− 0.01
$2\omega + \omega'$								
1	1	+ .001	. . .	+ .001	. .	+ .001	+ .001	. .
2	1	+ .035	. . .	+ .035	− .005	+ .030	+ .032	+ 0.03
3	1	+ .004	. . .	+ .004	− .001	+ .003	+ .004	. .
1	0	+ .002	. . .	+ .002	. .	+ .002	. .	. .
2	0	+ .001	. . .	+ .001	+ .001	+ .002	. .	. .
$4\omega - \omega'$								
4	0	. .	. .	. .	− .001	− .001	. .	. .
3	−1	+ .003	. . .	+ .003	− .001	+ .002	+ .002	. .
4	−1	. .	. .	. .	+ .005	+ .005	+ .005	+ 0.01
5	−1	. .	. .	. .	+ .001	+ .001	. .	. .

TABLE IV.—*The Moon's Latitude*—Continued.

g	g'	$\sin I \sin (f+\omega)$	s	$\sin \beta$	$\beta - \sin \beta$	β *Hansen.*	β *Delaunay* (1).	β *Delaunay* (2).
$-3\omega'$		″	″	″	″	″	″	″
−1	−3	− .002	. .	− .002	. .	− .002	− .002	. .
0	−3	− .015	. .	− .015	. .	− .015	− .010	− 0.01
$5\omega - 2\omega'$								
4	−1	. .	. .	. .	+ .001	+ .001	+ .003	. .
5	−1	. .	. .	. .	+ .001	+ .001	+ .002	. .
2	−2	+ .004	. .	+ .004	. .	+ .004	+ .002	. .
3	−2	− .089	. .	− .089	+ .024	− .065	− .068	− 0.07
4	−2	− .060	. .	− .060	− .186	− .246	− .246	− 0.25
5	−2	− .008	. .	− .008	− .137	− .145	− .142	− 0.14
6	−2	. .	. .	. .	− .030	− .030	− .028	− 0.03
7	−2	. .	. .	. .	− .004	− .004	− .003	. .
3	−3	− .004	. .	− .004	+ .001	− .003	− .003	. .
4	−3	− .004	. .	− .004	− .008	− .012	− .011	− 0.01
5	−3	. .	. .	. .	− .009	− .009	− .008	− 0.01
6	−3	. .	. .	. .	− .002	− .002	− .001	. .
$4\omega - 5\omega'$								
1	−5	+ .001	. .	+ .001	. .	+ .001	. .	. .
2	−5	− .001	. .	− .001	. .	− .001	− .002	. .
3	−5	+ .001	. .	+ .001	. .	+ .001	− .004	. .
$6\omega - 5\omega'$								
4	−5	− .005	. .	− .005	. .	− .005	. .	. .
5	−5	− .002	. .	− .002	. .	− .002	+ .001	. .
$7\omega - 6\omega'$								
4	−6	+ .031	. .	+ .031	. .	+ .031	+ .011	+ 0.01
5	−6	+ .060	. .	+ .060	. .	+ .060	+ .020	+ 0.02
6	−6	+ .043	. .	+ .043	. .	+ .043	+ .012	+ 0.01
7	−6	+ .014	. .	+ .014	. .	+ .014	+ .002	. .
8	−6	+ .002	. .	+ .002	. .	+ .002	. .	. .
4	−7	+ .004	. .	+ .004	. .	+ .004	. .	. .
5	−7	+ .010	. .	+ .010	. .	+ .010	. .	. .
6	−7	+ .008	. .	+ .008	. .	+ .008	. .	. .
7	−7	+ .001	. .	+ .001	. .	+ .001	. .	. .
$7\omega - 4\omega'$								
4	−4	− .001	. .	− .001	. .	− .001	. .	. .
5	−4	− .001	. .	− .001	− .004	− .005	− .002	. .
6	−4	. .	. .	. .	− .006	− .006	− .002	. .
7	−4	. .	. .	. .	− .003	− .003	. .	. .
$3\omega + 2\omega'$								
2	2	. .	. .	. .	− .001	− .001	+ .001	. .
3	2	. .	. .	. .	+ .002	+ .002	+ .002	. .
5ω								
4	0	. .	. .	. .	+ .001	+ .001	+ .002	. .
5	0	. .	. .	. .	+ .006	+ .006	+ .006	+ 0.01
6	0	. .	. .	. .	+ .002	+ .002	+ .002	. .

Table V.—*The Moon's Parallax.*

g	g'	$\frac{D(1+e\cos f)}{a(1-e^2)}$	Pert.	*Hansen's* sine Parallax.	*Delaunay's* sine Parallax. (1)	*Delaunay's* sine Parallax. (2)	D_2-D_1	$H-D_2$	*Adams* sine Parallax.
		"	"	"	"	"			"
0	0	3399.682	+ 22.405	3422.09	3422.7	3422.7	0	− 61	3422.32
1	0	+ 186.547	− .064	+ 186.483	+ 186.587	+ 186.55	− 4	− 7	+ 186.51
2	0	+ 10.220	− .059	+ 10.161	+ 10.198	+ 10.20	0	− 4	+ 10.17
3	0	+ .627	− .007	+ .620	+ .631	+ 0.63	0	− 1	+ 0.63
4	0	+ .040	. .	+ .040	+ .041	+ 0.04	0	0	+ 0.04
5	0	+ .003	. .	+ .003	+ .003	. .	.	.	. .
−4	−1	− .001	. .	− .001	. .	. .	.	.	. .
−3	−1	− .007	− .003	− .010	− .006	− 0.01	0	0	. .
−2	−1	− .067	− .055	− .122	− .092	− 0.09	0	+ 3	− 0.10
−1	−1	− .304	− .657	− .961	− .912	− 0.93	+ 2	+ 3	− 0.95
0	−1	− .018	− .375	− .393	− .427	− 0.43	0	− 4	− 0.40
1	−1	+ .299	+ .845	+ 1.144	+ 1.052	+ 1 11	+ 6	+ 3	+ 1.16
2	−1	+ .082	+ .067	+ .149	+ .103	+ 0 10	0	+ 5	+ 0.12
3	−1	+ .009	+ .003	+ .012	+ .006	+ 0.01	0	0	. .
4	−1	+ .001	. .	+ .001	. .	. .	.	.	. .
−1	−2	− .003	− .007	− .010	− .010	− 0.01	0	0	. .
0	−2	. .	− .008	− .008	− .012	− 0.01	0	0	. .
1	−2	+ .003	+ .009	+ .012	+ .013	+ 0.01	0	0	. .
2	−2	+ .001	. .	+ .001	. .	. .	.	.	. .
$2\omega-2\omega'$									
0	0	+ .001	. .	+ .001	. .	. .	.	.	. .
1	0	. .	− .021	− .021	− .013	− 0.01	0	+ 1	. .
2	0	− .001	− .001	− .002	. .	. .	.	.	. .
−1	−1	− .001	. .	− .001	+ .001	. .	.	.	. .
0	−1	+ .010	− .012	− .002	+ .002	. .	.	.	. .
1	−1	+ .010	− .237	− .227	− .379	− 0.38	0	− 15	− 0.23
2	−1	− .015	− .286	− .301	− .328	− 0.33	0	− 3	− 0.31
3	−1	− .015	− .034	− .049	− .040	− 0.04	0	+ 1	. .
4	−1	− .002	− .002	− .004	− .002	. .	.	.	. .
−3	−2	− .002	. .	− .002	. .	. .	.	.	. .
−2	−2	− .018	+ .004	− .014	− .008	− 0.01	0	0	. .
−1	−2	− .213	+ .092	− .121	− .101	− 0.10	0	+ 2	− 0.12
0	−2	− 2.128	+ 1.826	− .302	− .277	− 0.25	0	+ 2	− 0.31
1	−2	− .992	+ 35.301	+ 34.309	+ 34.166	+ 34.29	+ 12	+ 2	+ 34.30
2	−2	+ 1.990	+ 26.235	+ 28.225	+ 28.179	+ 28.20	+ 2	+ 3	+ 28.23
3	−2	+ 1.190	+ 1.894	+ 3.084	+ 3.064	+ 3.07	+ 1	+ 1	+ 3.09
4	−2	+ .154	+ .129	+ .283	+ .271	+ 0.27	0	+ 1	+ 0.28
5	−2	+ .015	+ .008	+ .023	+ .018	+ 0.02	0	0	. .
6	−2	+ .001	. .	+ .001	. .	. .	.	.	. .
−2	−3	− .001	. .	− .001	. .	. .	.	.	. .
−1	−3	− .008	+ .004	− .004	− .003	. .	.	.	. .
0	−3	− .094	+ .075	− .019	− .013	− 0.01	0	+ 1	. .
1	−3	− .069	+ 1.516	+ 1.447	+ 1.452	+ 1.47	+ 2	− 2	+ 1.45
2	−3	+ .091	+ 1.829	+ 1.920	+ 1.876	+ 1.91	+ 3	− 1	+ 1.92
3	−3	+ .082	+ .147	+ .229	+ .197	+ 0.22	+ 2	1	+ 0.22
4	−3	+ .012	+ .010	+ .022	+ .012	+ 0.01	0	+ 1	. .
5	−3	+ .001	. .	+ .001	. .	. .	.	.	. .

Table V.—*The Moon's Parallax*—Continued.

		$\frac{D(1 - e \cos f)}{a(1 - e)}$	Pert.	*Hansen's* sine Parallax.	*Delaunay's* sine Parallax. (1)	*Delaunay's* sine Parallax. (2)	$D_2 - D_1$	H − D.	*Adams'* sine Parallax.
2ε	2ω		″	″	″	″			″
0	−4	− .003	+ .002	− .001	. .	. .	.	.	. .
1	−4	− .004	+ .053	+ .049	+ .045	+ 0.04	0	+ 1	+ 0.05
2	−4	− .003	+ .089	+ .092	+ .076	+ 0.10	+ 2	− 1	+ 0.09
3	−4	+ .001	+ .008	+ .012	+ .005	+ 0.01	0	0	. .
4	−4	+ .001	. .	+ .001	. .	. .	.	.	. .
1	−5	. .	+ .001	+ .001	. .	. .	.	.	. .
2	−5	. .	+ .004	+ .004	. .	. .	.	.	. .
4ω − 4ε									
2	−3	. .	− .004	− .004	− .007	− 0.01	0	0	. .
3	−3	. .	− .006	− .009	− .008	− 0.01	0	0	. .
4	−3	. .	− .004	− .004	− .002	. .		.	. .
1	−4	− .002	+ .010	+ .008	+ .004	. .	.	.	. .
2	−4	− .005	+ .377	+ .372	+ .310	+ 0.31	0	+ 6	+ 0.37
3	−4	+ .022	+ .577	+ .599	+ .499	+ 0.50	0	+ 10	+ 0.60
4	−4	+ .030	+ .231	+ .261	+ .196	+ 0.20	0	+ 6	+ 0.26
5	−4	+ .012	+ .031	+ .043	+ .019	+ 0.02	0	+ 2	. .
6	−4	+ .002	+ .002	+ .004	. .	. .	.	.	. .
2	−5	.001	+ .033	+ .032	+ .016	+ 0.02	0	+ 1	. .
3	−5	− .002	+ .067	+ .069	+ .030	+ 0.03	0	+ 4	+ 0.06
4	−5	+ .004	− .031	+ .035	+ .011	+ 0.01	0	+ 2	. .
5	−5	+ .002	+ .005	+ .007	. .	. .	.	.	. .
2	−6	. .	+ .002	+ .002	.	. .	.	.	. .
3	−6	. .	+ .004	+ .004	. .	. .	.	.	. .
4	−6	. .	+ .002	+ .002	. .	. .	.	.	. .
6ω − 6ε									
3	−6	. .	+ .004	+ .004	. .	. .	.	.	. .
4	−6	. .	+ .010	+ .010	. .	. .	.	.	. .
5	−6	. .	+ .007	.007	. .	. .	.	.	. .
6	−6	. .	+ .002	+ .002	. .	. .	.	.	. .
4	−7	. .	+ .001	+ .001	. .	. .	.	.	. .
5	−7	. .	+ .001	+ .001	. .	. .	.	.	. .
2ω									
1	1	. .	− .003	− .003	− .002	. .	.	.	. .
2	1	. .	+ .001	+ .001	+ .002	. .	.	.	. .
−1	0	+ .002	− .002	0	. .	. .	.	.	. .
0	0	+ .035	− .035	0	. .	. .	.	.	. .
1	0	. .	− .709	− .709	− .708	− 0.71	0	0	− 0.71
2	0	− .039	+ .027	− .012	− .006	− 0.01	0	0	. .
3	0	− .002	− .002	0	. .	. .	.	.	. .
1	−1	. .	+ .002	+ .002	+ .002	. .	.	.	. .
2	−1	. .	+ .001	+ .001	+ .002	. .	.	.	. .
2ω′									
−1	3	+ .001	− .004	− .003	− .002	. .	.	.	. .
0	3	. .	− .007	− .007	− .007	− 0.01	0	0	. .
1	3	− .001	− .001	− .002	− .003	. .	.	.	. .

TABLE V.—*The Moon's Parallax*—Continued.

s	s'	$\frac{D(1 + e\cos f)}{a(1 - e^2)}$	Pert.	*Hansen's* sine Parallax.	*Delaunay's* sine Parallax. (1)	*Delaunay's* sine Parallax. (2)	$D_2 - D_1$	$H - D_2$	*Adams'* sine Parallax.
$2\omega'$		″	″	″	″	″			″
−2	2	. .	−.004	−.004	−.006	−0.01	0	0	. .
−1	2	+.038	−.086	−.048	−.050	−0.05	0	0	. .
0	2	+.007	−.112	−.105	−.109	−0.11	0	0	−0.11
1	2	−.036	−.047	−.083	−.082	−0.08	0	0	−0.09
2	2	−.006	−.003	−.009	−.008	−0.01	0	0	. .
3	2	−.001	. .	−.001	. .	. .	.	.	. .
−1	1	−.001	+.001	0	+.001	. .	.	.	. .
0	1	. .	+.001	+.001	+.001	. .	.	.	. .
1	1	+.001	. .	+.001	+.001	. .	.	.	. .
$4\omega - 2\omega'$									
2	−2	. .	−.014	−.014	−.015	−0.01	0	0	. .
3	−2	−.001	−.010	−.011	−.011	−0.01	0	0	. .
4	−2	−.001	. .	−.001	. .	. .	.	.	. .
$2\omega - 4\omega'$									
1	−4	. .	+.001	+.001	+.003	. .	.	.	. .
2	−4	. .	+.002	+.002	+.004	. .	.	.	. .
$\omega - \omega'$									
−1	0	−.001	−.002	−.003	−.002	. .	.	.	. .
0	0	−.008	+.007	−.001	. .	. .	.	.	. .
1	0	. .	+.146	+.146	+.151	+0.15	0	0	+0.14
2	0	+.008	+.008	+.016	+.013	+0.01	0	0	. .
3	0	+.001	. .	+.001	+.002	.	.	.	. .
−2	−1	+.001	. .	+.001	. .	. .	.	.	. .
−1	−1	+.012	+.003	+.015	+.007	+0.01	0	+1	. .
0	−1	+.055	−.044	+.011	+.008	+0.01	0	0	+0.01
1	−1	−.004	−.049	−.953	−.938	−0.97	+3	−2	−0.95
2	−1	−.055	−.051	−.106	−.097	−0.10	0	+1	−0.11
3	−1	−.007	−.003	−.010	−.006	−0.01	0	0	. .
4	−1	−.001	. .	−.001	. .	. .	.	.	. .
1	−2	. .	−.004	−.004	−.002	. .	.	.	. .
2	−2	. .	−.001	−.001	+.001	. .	.	.	. .
$3\omega - 3\omega'$									
2	−2	. .	+.003	+.003	+.003	. .	.	.	. .
3	−2	. .	+.002	+.002	+.002	. .	.	.	. .
1	−3	. .	−.009	−.009	−.005	. .	.	.	. .
2	−3	−.001	−.036	−.037	−.020	−0.02	0	+2	. .
3	−3	−.002	+.005	+.003	+.016	+0.02	0	−2	. .
2	−4	. .	−.002	−.002	. .	. .	.	.	. .
3	−4	. .	+.001	+.001	+.002	. .	.	.	. .
$\omega + \omega'$									
1	1	. .	+.007	+.007	+.007	+0.01	0	0	. .
$\omega - 3\omega'$									
1	−3	. .	−.002	−.002	−.002	. . .	.	.	. .